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Assessing and Affording the Control of Flood Risk

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Abstract

Flood is a most serious hazard to life and property. Dams, dikes and levees are often designed to a fuzzy quantity (PMF). Probabilistic design is preferable but requires that hydrological data be translated into a local monoscopic flood probability distribution. This process introduces information that goes beyond the facts. The method of relative entropy with quantile constraints minimizes this information and has a practical approximation, which is used here.

Socio-economic optimization provides a design criterion that reflects the Societal Capacity to Commit Resources (SCCR) to sustainable risk reduction. Details of financing have an important influence on the design of flood control projects by socio-economic optimization; since future life risk must be discounted like finances, the interest rate and amortization period influence design decisively. The example of a city protected by a levee casts a light on the relative importance of the factors influencing the design.

1. Introduction

The design work of the civil engineer is sandwiched between that of others: the scientists who provide much of the data about loadings and materials, and those authorities who provide the performance requirements. The development of probability-based system analysis it has made it possible to integrate data and analysis, in essence producing a probabilistic description of the performance of a given design. What remains for probabilistic design fully to become reality is a probabilistic description of the performance requirements, in particular for resistance. Explained below is such a description derivable from the economics of human welfare. The design of particular dams, dikes or levees provides relatively simple applications.

It is difficult to assess the risk of flood and to design facilities such that the risk is appropriately small. Among the reasons, apart from the lack of objective measures of acceptable risk, are scarcity of data and unknown probability distributions. Flood control facilities have typically long design lives, spanning over many generations, which raises another difficulty: How can the burden of financing, including the costs of risk mitigation, be distributed fairly between generations?

Progress in risk assessment has recently provided some rationales that can assist the decision-maker to choose robust and defensible designs. An objective standard for risk assessment, the Societal Capacity to Commit Resources (SCCR), derived from welfare economics and supported by accurate statistical data, has been proposed by Nathwani et al. (2005)[1]. The SCCR directly leads to optimum risk levels (Pandey et al. 2006)[2]in the way briefly summarized in Section 3. A design flood can be derived from the SCCR by optimization. The process leads from human welfare to design flood in a given problem setting. Not surprisingly, there is no universally optimal PMF – there is high sensitivity of the optimum to the social, economic, financial and physical circumstances and the way they are modelled. This is shown by an example in Section 5.

To arrive at a defensible design it is a professional imperative to avoid making arbitrary assumptions as much as possible. It has long been customary to choose mathematical probability distribution functions for the uncertain variables and then fit them in some fashion to available data. However, there is little justification for the assumption that the extreme rare events of interest will arise from the same regime(s) that produced the data. Enough improbable events – "outliers" -- that often produce precisely those catastrophic results that design should have prevented – have occurred to demonstrate that ill-founded mathematical assumptions can be dangerous (Klemeš 1987, 2000, 2002). Instead it is suggested in Section 2 to follow a more cautious statistical path, using cross-entropy estimation in order to minimize the information that must be added to the information of the data to produce the design distributions (Lind and Solana 1990, Lind and Hong 1991, Solana-Ortega and Solana 2001).

Financing arrangements can significantly influence the optimum design. It is well established that risks to life and health should be discounted to net present values along with costs (Rackwitz 2003). When design lives are long, this poses a dilemma: Any financially workable rate of interest, if held constant, can trivialize future risks in comparison to initial monetary costs, indeed so seriously as to be morally repugnant and socially indefensible. Some suggestions to solve this problem that have recently emerged (Rackwitz 2003, Pandey 2003, Lind 2007) are also presented briefly In Section 4 and illustrated in Section 5.

2. Cross-entropy Estimation

Any uncertain quantity used in the analysis of flood risk can be considered as a random variable produced by a "black box," a *generator* of independent random numbers. The realizations of this random variable, some of which are available as data, may suggest how the generator works and how it can be modelled mathematically. Nevertheless, there is little evidence to support such assumptions, particularly above or below the range of the data -- just where values relevant to design are to be found (Klemeš 2000). For simplicity it is assumed in the following that the data have been processed to take account of all physical and statistical signs of time-dependent drift (e.g. from change of climate or land use) of the generator's parameters.

Suppose that we are given the output from such a generator of independent random numbers. Each number may be written on the back of a card. You turn over n of the cards selected at random or in the order they have been dealt to you: $\{x_1, x_2, \ldots, x_n\}$. Since the order of the cards is random, the probability that the next number produced is greater than x_n equals $1/(n+1)$ – the same as the probability that its rank order is any other number between 1 and $n+1$. The fact that you know n realizations is immaterial. True, you can calculate the mean, variance and so on, and even guess at a good mathematical function to fit the data, but you have no knowledge of the tail of the distribution other than that it is monotonic. A distribution function $G(x)$, to be plausible, must therefore satisfy $G(x_n) = n/(n+1)$.

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You can, however, always fit a distribution function $F(x)$ to the data by the method of least squares or maximum likelihood, etc. Such a function may well be good science, but for an engineer there is reason to be cautious if the highest value observed conflicts with the mathematics. An engineer who strives to minimize the arbitrariness introduced in design in a given situation – called *monoscopic* by Matheron (1989) --has another option that is more defendable: minimizing the expected (Shannon) information introduced in the estimation process over and above the information content of the sample. Further, among the distributions *G(x)* that satisfy $G(x_i) = i/(n+1),\ i = 1, \ldots, n$ there exists exactly one that minimizes the information content relative to *F(x);* it is found by cross-entropy estimation (Lind et al.1989).

In practice the analysis is much simpler if the sample is reasonably large and you are interested only in extreme high or low values, with return periods greater than covered by experience. If a good approximation to the best fit function *F(x)* is found by any common method such as least squares or the method of moments, then *G(x)* can be calibrated for values greater than x_n as

$$G(x) = 1 - (n+1)^{-1} F(x_n)^{-1} [1 - F(x)]. \tag{1}$$

The cross-entropy approach is not well known in hydrological and materials science practice. The cross-entropy method seeks to minimize the influence of belief, which can be important for design as shown in Section 5.

3. The SCCR

The *Societal Capacity to Commit Resources (SCCR)* to risk reduction in a sustainable manner is a quantity *KG/E* that can be derived from a social indicator, the Life Quality Index (LQI). The Life-Quality Index $E^K G$ is a composite of the healthy life expectancy at birth *E* and the Gross Domestic Product per person *G*. *E* and *G* have long been used to quantify the health and wealth of a nation. Both are reliably measured. The parameter *K* reflects the trade-off we place on consumption and the value we attach to length of life. Using economic data for selected major OECD countries (Canada, U.S., France, Germany, UK) the value of the parameter $K=5.0$ has been recommended for analysis of life-safety projects (Rackwitz 2005; Pandey et al. 2005).

The LQI model determines the maximum level of public expenditure that justifiably can be incurred in exchange for a small reduction in the risk of death and yet will improve the expected life-quality for all. This value can be considered as a fair estimate of the societal capacity to commit resources to risk reduction compatible with sustainable human development. Suppose a small portion of *G*, *dG*, is invested in implementing a project, program or regulation that affects the public risk and modifies the life expectancy by a small amount *dE*. The net benefit criterion requires that there should be a net increase in LQI = $E^K G$. Therefore K dE/E + dG/G must be positive, from which it follows that the *Societal Capacity to Commit Resources (SCCR)* to risk reduction equals -dG/dE = KG/E.

The SCCR reflects on decisions about risks to life and health, facilitating the management of all such risks in a consistent, ethical and cost-effective manner. *E* and *G* reflect the expected length of life in good health and the available wealth to choose among possibilities, two indispensable constituents of “human development,” characterized as a “process of enlarging people’s choices” (UNDP 1990).

4. Discounting Risk Flow and Cash Flow

Flood control structures are expected to have long lives, often hundreds of years. When the risks and costs that are expected in one accounting period are paid for in another period, it is necessary to account for the time value of money (e.g. Rackwitz 2003). Indeed, for consistency the interest rate as a function of time must be the same for risk flows and cash flows (e.g. Pate-Cornell 1984). But if the interest rate is constant and greater than zero, then risks in the distant future become trivialized. However, the financing of these projects does not go on indefinitely; sooner or later the cash flow ends and the books are closed at the *financing horizon*. After this time, there are still risks of loss of life and property, but these risks do not involve a cash flow to or from the structure. So, the consistency requirement does not constrain the discounting of risks to life beyond the financing horizon. On principle, our duty with respect to saving lives is the same to all generations, whether in the near or the far future. Before the financing horizon ordinary principles of discounting must apply, but after this time no further discounting is justifiable. The principle implies that risk events beyond the financing horizon should be valued as if they occurred at the financing horizon (Lind 2007).

It has been suggested that many people feel they would pay more for reducing risks to their children than to their grandchildren or their descendants. This sentiment does not conflict with the principle just cited, because the *effective* current rate of interest, while constant during the financing period, decreases hyperbolically with time after the financing horizon (Lind 2007). A similar trend has been proposed by other studies (Pandey and Nathwani 2003).

The general procedure of socio-economic optimization of an engineering facility is illustrated in Figure 1 and applied in the following example.

5. Example – Flood Control

Consider a city of 100,000 inhabitants, situated on a flood plane and protected by a levee. The city was flooded in the early 1900s with a loss of several thousand lives. The existing levee was built in 1955 to elevation 16.5 m. However, the city was flooded again already in 2006 with a loss of 240 lives. The material losses were estimated at $450M.

It is now planned to rebuild and raise the levee to the optimal flood level. The alternatives are characterized by their crest elevation H. We approximate the estimated present value of the cost of an alternative as $C = C(H) = a(H^3 – b^3)$, where $a=\$100,000$ and $b=13$ m are constants. The levees are to have a design life of N=200 year.

It is planned to complete the new levee at the beginning of 2010 (= year 0 in its 200-year design lifetime). The project is to be financed by taxes. The taxes will amortize an issue of dedicated 30-year municipal bonds at an estimated 2.0% above inflation. Thus, the burden of financing is limited by the financing horizon $T = 30$ years, approximately the present generation's average remaining lifespan (Lind 2007).

If the levee fails, the total loss including the cost of reconstruction is estimated to be $\boldsymbol{L}=(L_L,L_M) = (300$ *lives*, $\$400M+C)$. The risk vector is the expected value of $\boldsymbol{L}$ over the set of all outcomes. The risk is to be assessed against society's capacity to commit resources to reduce it. With $K=5$, $G=\$35,000$ per person per year and $E=78$ years, the SCCR equals $KG/E=\$2244$ per person per year[2]. Design alternatives are evaluated following the general scheme in Figure 1.

There are now n = 98 years of flood data, including the year 2006 when the estimated flood level reached 17.1 m. The seven highest observations x_i , i = 91, 92, . . . 98 are plotted in Figure 2 as (x_i, i/(1+n)). The annual flood level 1909-2006 had mean $m_Y = 6.99$ m and sample standard deviation $s_Y = 2.34$ m. There is no statistically significant time trend. Of course, in practice the hydrological data require critical hydrological review, considering the many geographical and meteorological factors involved. Here the data is assumed for the present purpose to faithfully represent all that is known about the conditions under which the new levee is to function. Among several candidate distribution types fitted by the method of moments, a Gumbel (Extreme-I) distribution fits the data best and is shown in Figure 2. The parameters are $\alpha = (\pi/6^{-1/2})/(2.34\ m) = 0.549\ m^{-1}$ and $u = m_Y - \gamma/\alpha = 5.939\ m$; here $\gamma = 0.577...$ is Euler's constant (Benjamin and Cornell 1970). $F(x) = EX_{I,L}(u,\alpha) = exp\{-exp[-\alpha(x-u)]\}$. Notice that several large floods, including the 2006 flood of 17.1 m, plot quite far from the fitted curve. So it is judged best to correct the upper tail above 17.1 m by minimizing cross-entropy (Section 2; see Lind and Solana (1990)).

Figure 2 shows that there have been several extremely large floods larger than the fitted Gumbel distribution, including the two that caused levee failure and several "near misses." The preferred cross-entropy approximation employs $F(x)$ as the reference (or "panscopic") distribution (Matheron 1989). However, in this application it is the "monoscopic" distribution $G(x)$ that is of interest. For $x > x_n = 17.1\ m$ – clearly the only range of interest here -- $G(x)$ can be written as

$$G(x) = 1\text{-}c[1\text{-}F(x)] = 1 - c(1 - \exp\{-\exp[-\alpha(x\text{-}u)]\}),\ x > x_n. \tag{2}$$

$G(x)$ is shown in heavy line in Figure 2. The constant c equals 4.62, calculated such that $1\text{-}G(x_n)=1/(1+n)$. The annual probability of flood levels above x_n is thus $c=4.62$ times higher than the conventional analysis indicates. With proper maintenance the annual probability of failure is constant, $p=!\text{-}G(H)$. For a levee of elevation H m, the conditional annual probability of flood is $p = 1 - G(H)$. The probability of surviving another year is $q=G(H)$. All cases can be analyzed in closed form.

The designers have several choices in the analysis. They consider (a) cross-entropy estimation (preferred) and conventional probability analysis; (b) a financing horizon T of 30 years (preferred) or 200 years, and (c) discounting at an annual rate r of 0%, 2% (preferred) or 3.5% above inflation. The annual discounting factor is $f = (1+r)^{-1}$.

The event tree in Figure 3 illustrates the calculation of the loss expectation. The probability that the levee survives its design life is $G^N(H)=q^{200}$, while the probability of failure during the design life is $1\text{-}q^{200}$. The probability of failure during the i-th year of service is $q^{i-1}p$, and the corresponding expected loss is $q^{i-1}p\mathbf{L}$. Its net present value is discounted to $f^i q^{i-1} p\ \mathbf{L}$ *if* $i \leq T$ and $f^T q^{i-1} p\mathbf{L}$ *if* $i>T$. The sum of all loss terms from $i = 1$ *to* T, the financing horizon, equals $fp\mathbf{L} + f^2 q^1 p\mathbf{L} + \ldots + f^T q^{T-1} p\mathbf{L} = fp\mathbf{L}(1\text{-}f^T q^T)/(1\text{-}fq)$. If $T=N$ this gives directly the total expected cost as

$$E(\mathbf{L}) = (0,C)+fp\mathbf{L}[1\text{-}(fq)^T]/(1\text{-}fq),\ \text{if } T=N; \tag{3}$$

if $T<N$, the sum of the loss terms for $i>T$ equals $p(fq)^T\mathbf{L}\ [1\text{-}q^{N-T+1}]/(1\text{-}q)$. Thus the net present value of the expectation of the total cost equals

$$E(\mathbf{L}) = (0,C)+\{(fp)[1\text{-}(fq)^T]/(1\text{-}fq)+(fq)^T(1\text{-}q^{N-T})\}\mathbf{L}. \tag{4}$$

Inserting trial values of $H = 17, 17.5, \ldots, 24$ and the values of all other constants gives $E(\mathbf{L}) = E(L_L,L_M)$ as a function of H. This generates a curve $L_M = f(L_L)$, one for each alternative analysis. As an example Figure 4 shows the curve of trial designs for the preferred analysis. The optimum design has the elevation H where the slope of the curve equals -SCCR. The calculations are conveniently done on a spreadsheet. For each year period from 2010 to 2209 the probabilities of survival and failure and expected losses are calculated, discounted to 2010 and summed. Table 1 gives the results.

6. Discussion

In an earlier study of socio-economic structural optimization of an office building, it was shown that the valuation of risk to human life and health can influence design, but that the influence may be small

(Lind 2005). In contrast Table 1 shows, first, that the optimal crest elevation (row 5) can differ by four meters, with construction cost (row 6) correspondingly differing by more than 100%, depending on the choice of hydrological and financial models. Still, for all alternatives the payments required to amortize the cost of the levee (row 7) are modest.

The lifetime failure probabilities (row 10), computed on the basis of the monoscopic distribution G(x), vary correspondingly from 13 % to 51 %. These failure probabilities may seem rather high (Compare Shalaby (1994)), but they are distributed over six generations of the population. The loss of life expectancy (row 9) is negligible, varying from a couple of hours to about one third of a day.
Disregarding Alternative 4 that uses a conventionally fitted but inappropriate hydrological model, the results show the importance of interest rates and financial time horizon; this sensitivity appears to be a general feature of risk-based optimization. On the other hand, the results are rather less sensitive to the SCCR-value: SCCR can be increased by 3 % or reduced by 29 % without changing the optimum of the preferred alternative by more than 0.5 m.

Figure 4 would suggest that the optimum is rather flat, such that not much is lost if a design differs from the optimal. On the assumption that Alternative 0 is optimal, the last five rows of Table 1 show the expected losses. Row 11 shows the expected value of the loss of life over the 200-year design lifetime. In the case of Alternative 3 there is an expected saving of lives, which of course comes at the expense of higher construction costs; this saving of lives is not a bargain, however – the excess funds could be used to better effect through other life-saving interventions, of which there are many. This is borne out in rows 12 and 13, where future losses are properly counted at their net present value. Equivalently, the deviations from optimum can be measured in terms of net present value of the total expected cost (rows 14 and 15). The deviations are not trivial, showing again the importance of the financing scheme in design. The flatness of the optimum depends on the attenuation rate of the failure probability and the rate of increase of construction costs with the height of the levee; if these values are typical, then the conclusion about the sensitivities would apply to similar projects.

7. Conclusions

As the hazard of floods continues to cause major losses of life and property, design to a “probable” maximum flood must yield to design based on more quantifiable measures of reality. Standard statistics applied to hydrological time series can lead to poor modelling of the extreme values that are of interest in design. One reason is that distribution assumptions introduce arbitrary information. Another is that low and central values of the data may unduly influence estimation of the upper tail. The paper explains and illustrates a simple approximation to quantile-constrained cross-entropy estimation that counters both sources of error.

To be useful as a rational basis for design, quantified probabilistic analysis requires quantified measures of acceptable or optimal risk. Fuzzy notions of keeping the risks “As Low As Reasonably Achievable (or Practicable)” (ALARA(P)) or the like are unsuited for quantified risk analysis for many reasons, including the need for consistency in the management of public funds. The societal capacity to commit resources to risk reduction is strictly limited. Enormous resources can be, and are, allocated to mitigate many manageable risks to life and health. However, the economics of human welfare sets a well-defined limit, the SCCR, to the funds that sustainably can be allocated to mitigate public risks. As illustrated here, flood control provides a context where the societal capacity to commit resources enters design in a simple way.

Civil engineering facilities have typically long design lives, and so the discounting of future risks, whether to life and property, is of decisive importance for design. Financial instruments allow public funding to be allocated continuously over time to risk reduction interventions. Accordingly, the time series of projected risk, the risk flows, must be discounted like projected cash flows. Such discounting is limited for each project to the period of financing. Together with interest rates, the financing horizon is important in civil engineering design.

In practice there are many considerations beyond formal societal optimality that must be taken into account in design. Indeed, “The greatest uncertainties and risks in water resources systems arise from the fact that water resources are often used as proxies over which political battles . . . are waged.” (Klemeš 2002). In this arena socio-economic optimization provides a dispassionate solution to the problem of arbitrariness in design. Flood control provides an important transparent application of the optimization of facilities in the public interest.

8. Acknowledgments

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9. References

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Table 1. Synopsis of Alternative Designs.

1	Analysis Alternative:	0	1	2	3	4
2	Flood Height Distribution	G(x)	G(x)	G(x)	G(x)	F(x)
3	Financing Horizon, years	30	30	200	200	30
4	Real annual Interest, %pa	2	3.5	2	0	2
5	Design: Crest Elevation, m	20.5	20	19	22	18
6	Construction Cost, $M	642	580	466	845	364
7	Annual amortization pmt/person, $	285	313	207	42	161
8	Gross Mortality increment, /M	4.7	6.2	10.7	2.1	18.5
9	Exp. LoL Expectancy, hours/person	3.3	4.3	7.5	1.4	12.9
10	Lifetime Probability of Failure	0.27	0.34	0.51	0.13	0.71
11	Expected LOL over optimal, $M	0	21	72	-42	132
12	Expected NPV LOL over optimal, $M	0	4	22	12	48
13	do. in % of optimal	0%	9%	47%	26%	102%
14	Expected Cost over optimal, $M	0	13	78	43	124
15	do. in % of $642M	0	2.0	12	7	19

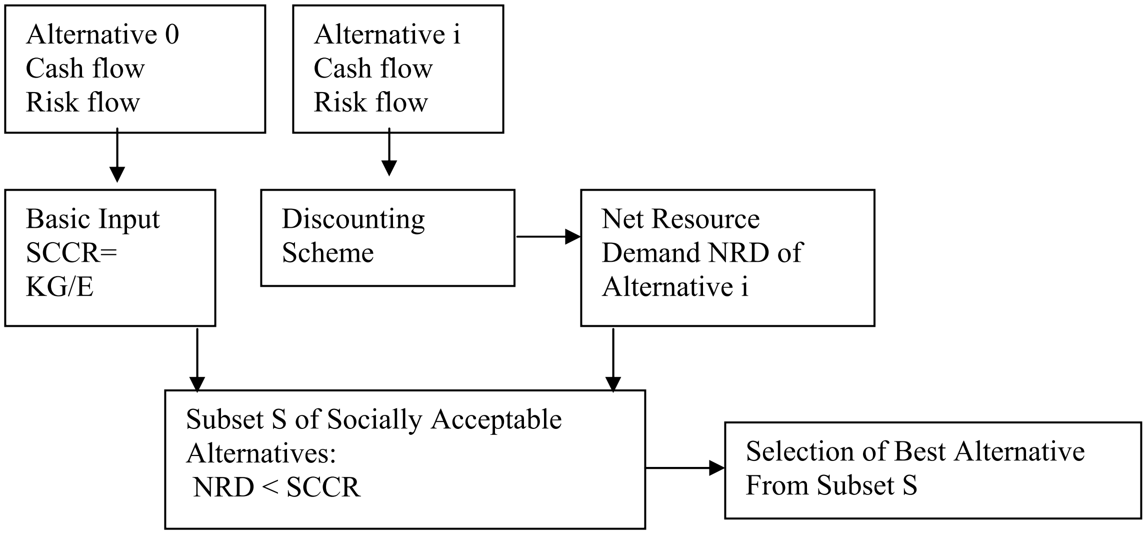

Figure 1. Typical Scheme of LQI Analysis in Civil Engineering.

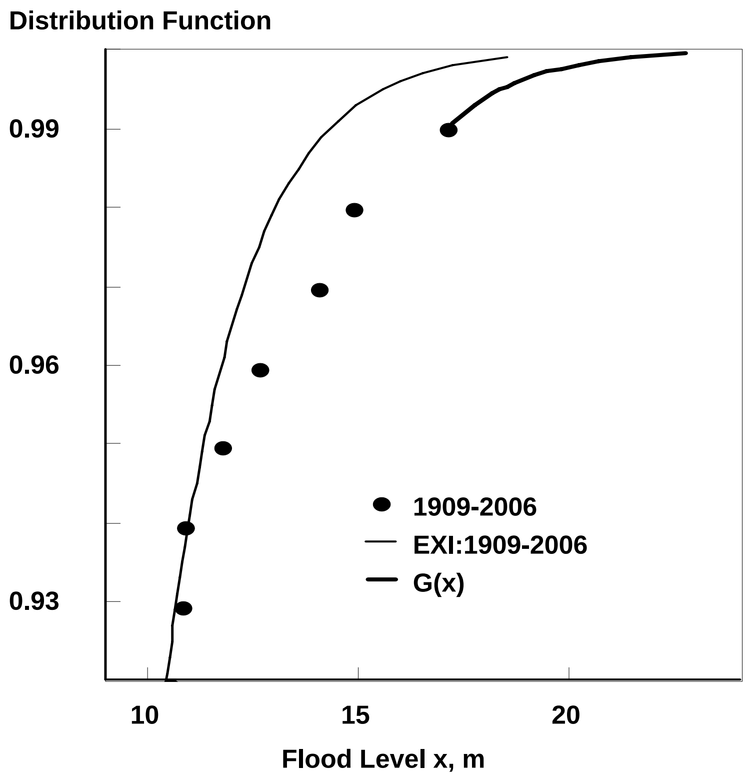

Figure 2. Plot of the most extreme flood levels observed

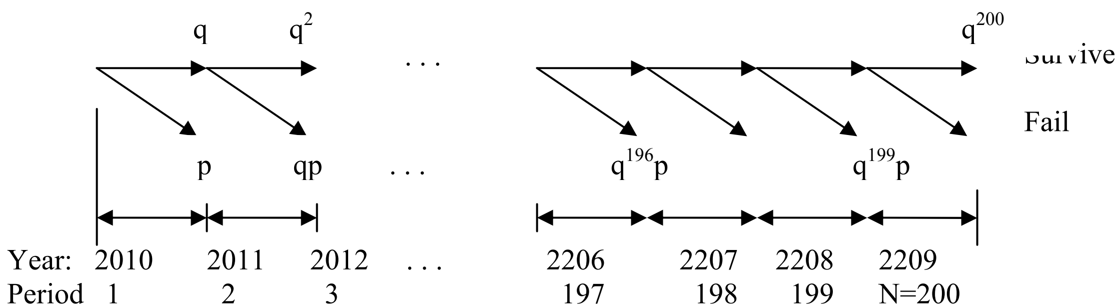

Figure 3. Event tree for Survival or Failures of the Levee.

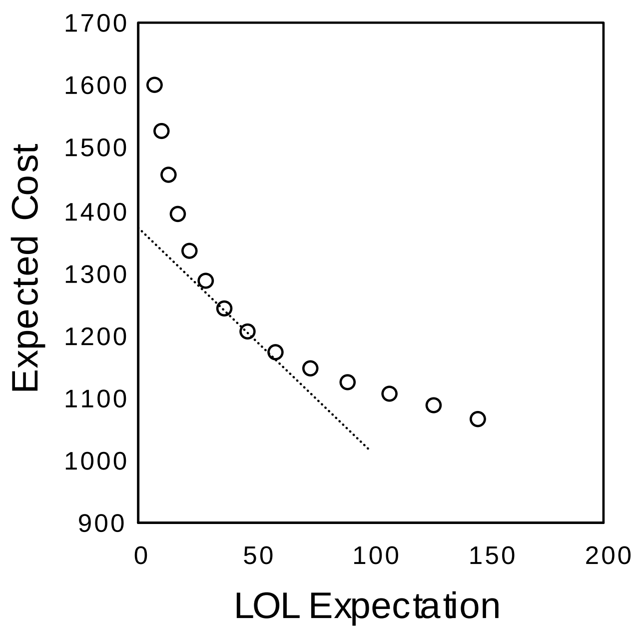

Figure 4. Expected cost vs. Total Expected Fatalities for Alternative 0