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Unified Reliability and Design Optimization in Earthquake Engineering

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Abstract

In this paper, a methodology is put forward that takes advantage of the emerging focus on structural damage in earthquake engineering. Contrary to the traditional focus on collapse limit-states in reliability analysis, a versatile "unified" limit-state formulation is employed. The methodology underscores the advantages of formulating explicit probabilistic models for the ground motion, structural response, damage, and ensuing losses. Next, by formulating limit-state functions in terms of total cost – including both the cost of construction and damage – a novel reliability-based design optimization procedure is put forward. By carrying out this design optimization in the tail of the total cost distribution; at unlikely but potentially devastating losses, an optimization strategy akin to the use of "risk averse" utility functions in classical decision analysis is achieved.

1. Introduction

The objective in this paper is twofold; to augment traditional structural reliability analysis with damage and cost models – here called a "unified" reliability formulation – and to utilize the unified reliability analysis in a novel design optimization approach that extends the traditional "expected cost" minimization. The developments are regarded as an extension of the current structural engineering practice, in which design codes have a prominent position. The design codes serve the important purpose of ensuring life safety, that is, prevent collapse, in extreme loading events, such as earthquakes. However, the code-oriented practice suffers the weakness that damage lesser than what causes collapse is not explicitly exposed. In contrast, the repair cost, business interruption, and occupancy prevention associated with damage may pose significant concerns to owners, occupants, and other stakeholders. This motivates the present study, where damage and loss limit-states are considered instead of the traditional collapse considerations.

It is argued herein that the utilization of reliability methods in damage and loss predictions is particularly appropriate. While structural collapses are often associated with human error and unanticipated effects, damage during earthquakes is purposely implied by design codes to dissipate seismic energy. Hence, it is not the occurrence of damage that is in question, but the level of damage. Moreover, the prediction of structural response and damage cannot be made in a deterministic manner, although numerical simulation models have improved dramatically in recent decades. A number of uncertainties are present even for prescribed ground motions due to uncertainties associated with the prediction of structural response, damage, repair costs, loss, downtime, etc. Based on these considerations, the case is here made that damage-based limit-states along with sophisticated inelastic structural models represent a renaissance for structural reliability methods. In particular, the awareness among engineers, stakeholders, and policymakers of the reliability methodology carries the potential of improving the current non-informative code-based engineering practice, and supplementing the early attempts on performance-based earthquake engineering described in the following.

Inclusion of damage and loss models is already being contemplated by the research community. Efforts by the Pacific Earthquake Engineering Research Center (PEER) have been particularly prominent in the field of earthquake engineering. The PEER methodology, originally proposed by Cornell and Krawinkler (2000) is based on disaggregating the problem into four models: the seismic model that predicts the intensity measure(s) *IM*, the structural model that predicts the structural response(s) referred to as engineering demand parameter(s) *EDP*, the damage model that predicts the damage measure(s) *DM*, and finally the loss model that predicts the decision variable(s) *DV*. Importantly, the decision variable(s) represent loss, downtime, or other measures of direct concern to decision makers.

Within the PEER community it is recognized that the performance predictions must be made in a probabilistic manner. In fact, the prediction models are developed to produce conditional probabilities. Specifically, each model provides the probability that the output measure exceeds a threshold, given a value of the input measure. This is a conditional complementary CDF, where CDF denotes a cumulative distribution function. For example, the damage model provides the conditional complementary CDF $G(dm|edp) \equiv \mathrm{P}(DM > dm | EDP = edp)$, where upper-case symbols denote random variables and lower-case symbols denote the associated realizations. The corresponding conditional probability density function (PDF) is $f(dm|edp) = |\mathrm{d}G(dm|edp)/\mathrm{d}(dm)|$, where absolute value sign is employed because the slope of G is negative.

In an assumption that will later be generalized the variables *IM*, *EDP*, *DM*, and *DV* are first taken to be scalars. This allows the straightforward use of the theorem of total probability three times to evaluate the complementary CDF of the decision variable *DV* (Cornell and Krawinkler, 2000):

$$\begin{aligned} G(dv) &= \int_0^\infty \int_0^\infty \int_0^\infty G(dv\,|\,dm) \left| \frac{\mathrm{d}G(dm\,|\,edp)}{\mathrm{d}(dm)} \right| \left| \frac{\mathrm{d}G(edp\,|\,im)}{\mathrm{d}(edp)} \right| f(im)\ \mathrm{d}(im)\mathrm{d}(edp)\mathrm{d}(dm) \\ &= \int_0^\infty \int_0^\infty \int_0^\infty G(dv\,|\,dm) \left| \mathrm{d}G(dm\,|\,edp) \right| \left| \mathrm{d}G(edp\,|\,im) \right| f(im)\ \mathrm{d}(im) \end{aligned} \qquad (1)$$

Several recent publications address the evaluation performance probabilities in the context of Eq. (1). Moehle *et al.* (2005) and Yang (2006) present an example application in which nonlinear dynamic structural analyses, damage models, and loss models are employed to obtain the probability of various levels of loss; effectively obtaining $G(dv)$. A simplified graphical approach to evaluate Eq. (1), termed "Fourway," is suggested by Mackie and Stojadinovic (2006). The Fourway approach allows approximate graphical considerations in lieu of refined analysis. In this paper, an alternative to the aforementioned approaches is put forward. The unified analysis that is put forward in this paper represents an alternative approach to address the problem in Eq. (1). In addition to the advantages generated by using a reliability formulation, the proposed approach facilitates novel design optimization that explicitly addresses the unlikely but potentially devastating outcomes in the tail of the distribution of $G(dv)$.

2. Unified Reliability Analysis

The basis for the reliability approach is the utilization of underlying probabilistic models instead of intermittent conditional probabilities. That is, rather than solely utilizing fragility curves, the fundamental probabilistic models are explicitly articulated. This is appealing for several reasons, including the explicit account of uncertainties and the facilitation of reliability analysis. Recent research supports these developments by developing multifaceted probabilistic models for member capacities and post-damage response (Gardoni et al. 2002; Zhu et al. 2007). However, it is strongly emphasized that the approach proposed herein by no means implies a criticism of the procedure put forward by Moehle *et al.* (2005) and Yang (2006). Both approaches have unique advantages. While the approach presented in the following is believed to represent a consistent and versatile framework to account for uncertainties, it requires reliability analysis tools and explicit probabilistic models that for some contemporary engineers are novel concepts.

The unified reliability analysis takes advantage of damage and cost models to extend the traditional structural reliability formulation. As an introduction to the methodology, consider the component reliability problem (Ditlevsen and Madsen 1996)

$$p = \int \underset{g(\mathbf{r})\leq 0}{\cdots} \int f(\mathbf{r})\mathrm{d}\mathbf{r} \tag{2}$$

where p is the sought probability, $\mathbf{r}$ is the vector of random variables, $f(\mathbf{r})$ is the joint probability density function for the random variables, and $g(\mathbf{r})$ is the limit-state function that defines the event for which the probability is sought. $g(\mathbf{r})$ is defined so that it takes on a negative value for realizations of $\mathbf{r}$ that yields the event of interest. Consequently, Eq. (2) represents the integration of the joint probability density $f(\mathbf{r})$ over the domain in the outcome space of $\mathbf{r}$ where $g(\mathbf{r})$ is negative.

Although Eq. (2) cannot be solved analytically in the context of finite element reliability analysis, several reliability methods are available to solve it numerically. These include the first and second-order reliability methods (FORM and SORM), sampling methods, and response surface methods (Ditlevsen and Madsen 1996). It is asserted in this paper that these methods have reached a level of maturity and accessibility that they can be considered part of a structural analyst's standard tools, in the same manner as finite element software has become. This paper extends the finite element reliability methodology by defining limit-state functions in terms of the decision variables, *DV*. To this end, consider the limit-state function

$$g(\mathbf{r}) = dv - DV = dv - DV(\mathbf{r}, DM(\mathbf{r}, EDP(\mathbf{r}, IM(\mathbf{r})))) \tag{3}$$

where dv is a threshold defined by the analyst. The dependence of the decision variable DV on the other variables in the problem is explicitly shown in the last term. Notably, elements of the vector $\mathbf{r}$ may enter in any of the models, as noted below. Effectively, a reliability analysis with this limit-state function in Eq. (3) produces the probability that the decision variable DV exceeds the threshold dv. Fig. 1 shows a schematic illustration of the evaluation of DV during the reliability analysis. The reliability module seeks the value of the decision variable DV (or vector of decision variables, $\mathbf{DV}$) for a given realization of the vector of random variables, $\mathbf{r}$. It is emphasized by boldface symbols in Fig. 1 that the input and output from each model could be vector-valued. The prospect of including vector-valued measures is significant for practical applications because it provides flexibility in defining hazards, structural responses, damage, and loss estimates.

Additional significance of the reliability approach is apparent by noting that it addresses the triple integral in Eq. (1). Specifically, by performing a reliability analysis with the limit-state function in Eq. (3) for different thresholds dv the resulting probability $p(dv)$ is akin to $G(dv)$ from Eq. (1). Notably, the unified reliability analysis circumvents the evaluation of conditional probabilities, such as $G(dm|edp)$ in the triple-integral formulation in Eq. (1). This readily facilitates vector-valued measures, as noted above. Also, the reliability analysis results exhibits accuracy in the tail of the resulting probability distribution, and, as noted in Fig. 1; any of the random variables $\mathbf{r}$ may be utilized by any of the four models, which is contrary to the fundamental assumption in the multiple use of the theorem of total probability to arrive at the triple-integral in Eq. (1).

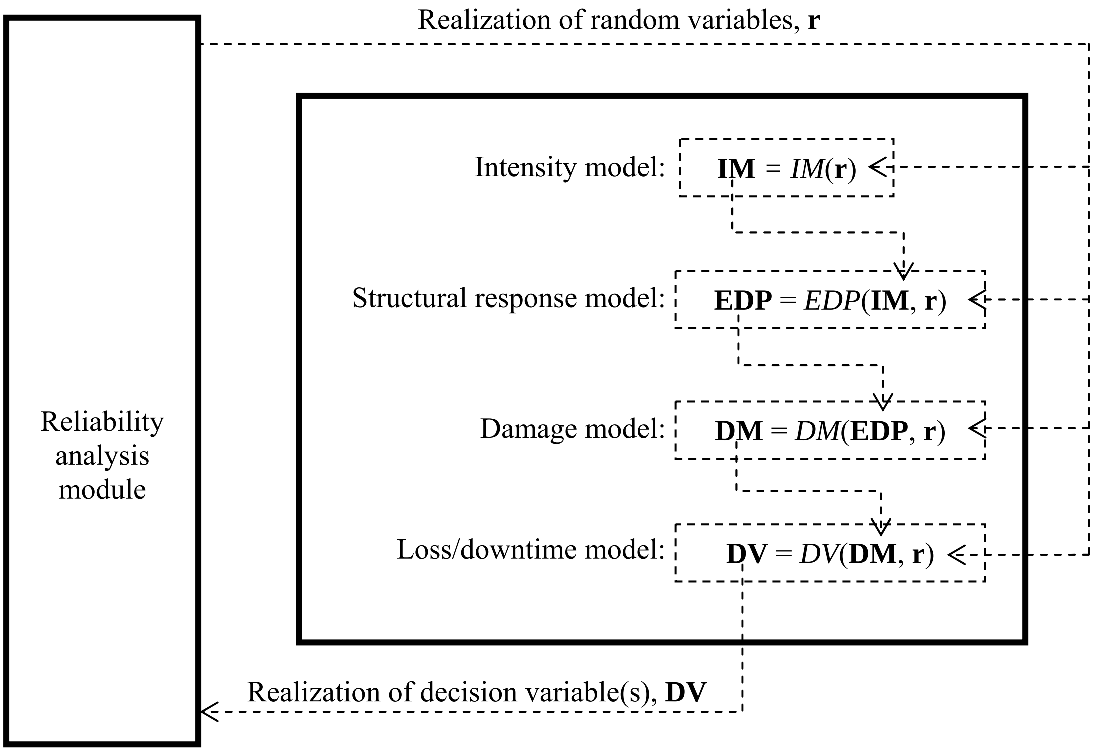

Figure 2: Evaluation of the decision variable(s) in unified reliability analysis.

In summary, by varying the threshold *dv* in the limit-state function in Eq. (3) the unified reliability analysis produces the same conceptual result as the procedure outlined in the previous section. FORM is an attractive method to solve the reliability problem, due to its limited number of evaluations of the limit-state function, which usually is a computationally costly matter. Moreover, FORM generates parameter importance measures that provide physical insight into the sources of earthquake-induced damage (Haukaas and Der Kiureghian 2005). On the other hand, FORM requires a continuously differentiable limit-state function. Under certain circumstances this may cause difficulties that smoothing techniques have proven promising to remedy (Koduru and Haukaas 2007a). That reference, and (Koduru and Haukaas 2007b), demonstrate the presented methodology by means of real-scale sophisticated structural models, comprehensive damage computations, and probabilistic ground motions.

3. Design Optimization

In this paper, a novel design optimization methodology is put forward that is facilitated by the unified reliability formulation. The developments are motivated by the recognition that the ultimate objective in structural engineering is to achieve designs that comprise an optimum balance between cost and safety. Historically, structural designs have been based on experience, judgment, and relatively straightforward equilibrium considerations. In recent decades the use of design codes in conjunction with the load and resistance factor design approach has formalized the utilization of safety factors. By “code calibration” the safety factors are tuned to the safety that is implied by the currently accepted engineering practice. However, this practice is largely focused on strength limit-states and not performance such as damage, downtime, and cost that is of more direct concern to stakeholders. Moreover, the traditional approach does not necessarily imply that the optimal design – from a cost-benefit standpoint – is achieved.

In recent years, the advent of numerical simulation models and robust reliability methods has led to an increased use of design optimization techniques. Notably, the merger of expected cost based objective functions and reliability methods to assess the structural safety addresses the problem of achieving the optimum balance between cost and safety. This methodology, termed “reliability-based design optimization” (RBDO) (Enevoldsen and Sorensen 1994; Kirjner-Neto *et al.* 1998; etc.) is subscribed to in this paper, where the total cost – including construction cost and cost of damage in impending earthquake(s) – is adopted as the rational basis for structural design optimization. Naturally, the unified reliability analysis presented earlier will serve as an important ingredient in the proposed methodology.

Traditionally, RBDO is performed by minimizing the total expected cost. As an illustration, consider the abstract example of a new design that is associated with the construction cost $c_c(\mathbf{d})$, where $\mathbf{d}$ is the vector of design variables. The design variables are the parameters that define the design and are at the discretion of the engineer. Examples of design variables are structural dimensions and strengths, or more precisely their mean or other characteristic values that are at the discretion of the engineer. This assertion is made because most structural parameters are associated with uncertainty and the engineer cannot formally specify the *outcome* of an uncertain variable. Instead, by specifying, e.g., a certain concrete quality the engineer influences the mean concrete strength.

The total expected cost is defined as the sum of the construction cost (or retrofit cost) plus the expected cost of failure. The expected cost of failure – traditionally meaning collapse – is the product of the probability of failure $p_f(\mathbf{d})$ and the failure cost $c_f(\mathbf{d})$ discounted to the present time. Hence, the problem of minimizing the total expected cost reads

$$\mathbf{d}^* = \arg\min\left\{c_c(\mathbf{d}) + c_f(\mathbf{d})\, p_f(\mathbf{d})\right\} \tag{4}$$

where $\mathbf{d}^*$ denotes the optimum design. This problem formulation is usually augmented with constraints on the design variables and the failure probability to account for problem-specific concerns. A number of contributions have been made to the solution of Eq. (6), including Enevoldsen and Sorensen (1994), Madsen and Friis Hansen (1992), Kuschel and Rackwitz (2000), Gasser and Schueller (1998), and Royset et al. (2006), and Liang et al. (2007).

In the context of problems in which the decision maker faces different levels of potential loss, the above objective of minimizing the expected cost has been criticized. Specifically, the expected cost approach does not explicitly address the unlikely but potentially devastating costs that are in the tail of cost distribution (Haimes 1998). In other words, the focus on the expected cost (mean cost) is questioned. Haimes (1998) addresses this issue by separating the cost axis into intervals and applying a multi-objective technique to minimize the cost from each interval. An alternative approach is suggested in this paper that employs unified reliability analysis in conjunction with minimization in the tail of the cost distribution instead of at the expected cost. To this end, the problem formulation in Eq. (5) is adapted to consider damage limit-states instead of collapse limit-states.

Consider the limit-state function in Eq. (5) in which both the construction cost and the cost of damage is included:

$$g(\mathbf{d},\mathbf{r}) = c_o - c_c(\mathbf{d},\mathbf{r}) - c_d(\mathbf{d},\mathbf{r}) \tag{5}$$

where $\mathbf{r}$ is the vector of random variables, c_o is the cost threshold set by the analyst, $c_c(\mathbf{d},\mathbf{r})$ is the cost of construction (or retrofit), and $c_d(\mathbf{d},\mathbf{r})$ is the cost associated with damage. Notably, the construction cost and damage cost are, in general, both functions of the decision variables and the random variables. By varying the threshold c_o the probability distribution for the total loss is obtained; this is the essence of the previously presented unified reliability analysis.

As an introductory example of the proposed design optimization methodology – intended for pedagogical purposes – consider the exceedingly simple case of a high-rise building with one dominating response mode that is idealized into a one-degree-of-freedom problem. The problem is idealized in term of the equivalent stiffness k, the applied load P, and the roof-level displacement response $u=P/k$. In this example the building stiffness k is considered to be at the discretion of the engineer, while the load P is a normal random variable with mean μ_P=15,000kN and standard deviation σ_P=4,500kN. That is, a stiffness equal to k=75,000kN/m would give a displacement equals to 0.2m. However, the displacement causes damage and, thus, comes at a cost. The cost of damage is assumed to be proportional to the displacement: $c_d(P,k)=m_u \cdot P/k$, where m_u is equal to \$10,000 per cm. On the other hand, the cost associated with increasing the stiffness – effectively reducing the displacement and thus damage – is assumed to be proportional to the value of the stiffness: $c_c(k)=m_k \cdot k$, where m_k is equal to \$2 per kN/m. Consequently, the limit-state function in Eq. (3) is formulated as

$$g = c_o - c_c(\mathbf{d},\mathbf{r}) - c_d(\mathbf{d},\mathbf{r}) = c_o - m_k \cdot k - m_u \cdot \frac{P}{k} \tag{6}$$

From a reliability standpoint the limit-state function in Eq. (6) is exceedingly simple and amenable to second-moment analysis that yields the reliability index

$$\beta(k,c_o) = \frac{\mu_g}{\sigma_g} = \frac{c_o - m_k \cdot k - m_u \cdot \dfrac{\mu_P}{k}}{m_u \cdot \dfrac{\sigma_P}{k}} \tag{7}$$

where μ_g and σ_g denotes the mean and standard deviation of the limit-state function, respectively. The explicit dependence of the reliability on the cost threshold c_o and the design variable (stiffness) k is shown to ease the interpretation of subsequent plots. The probability that the total cost exceeds the threshold c_o is related to the reliability index by $p(k,c_o) = \Phi(-\beta(k,c_o))$, where Φ is the standard normal CDF. By varying the threshold c_o for the fixed stiffness value k=75,000kN/m the complementary CDF $p(k,c_o)$ and the associated PDF $\left|\mathrm{d}p(k,c_o)/\mathrm{d}c_o\right|$ of the total cost is obtained, as shown in Fig. 2.

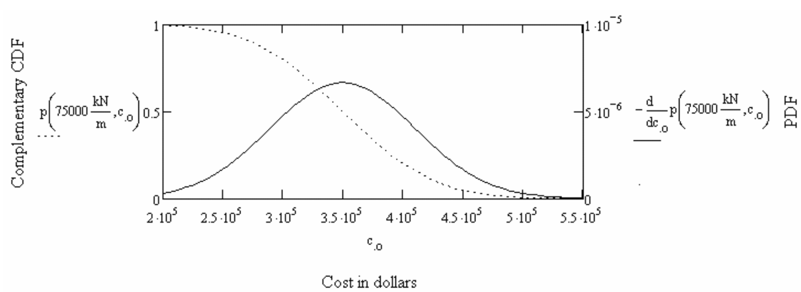

Figure 2: Probability distribution (complementary CDF and associated PDF) for the total cost, for a fixed value of the design variable (stiffness).

Relative to the previous section, Fig. 2 is the typical outcome of a unified reliability analysis. It is observed that the expected total cost is approximately \$350,000, while a cost in excess of half a million dollars is unlikely. Conversely, by keeping the load value fixed at its mean value and varying the design variable (stiffness) the cost variation shown in Fig. 3 is obtained. It is observed that a low stiffness will give high damage costs, c_d, while a high stiffness comes at a high construction cost, c_c. It is also seen in Fig. 3 that the total cost exhibits a u-shaped behaviour that indicates that a stiffness around 85,000kN/m will give the lowest total cost. For the deterministic problem, this demonstrates the optimization problem at hand.

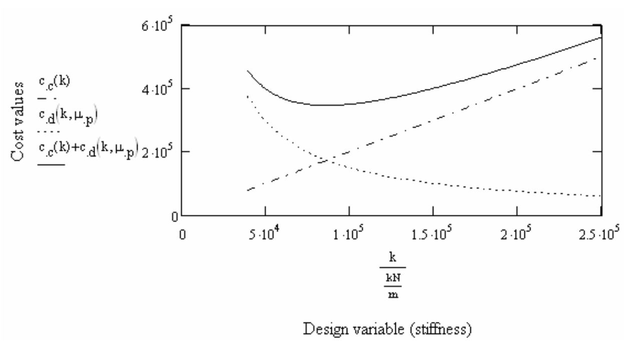

Figure 3: Construction cost and damage cost at the mean value of the random load, as function of the design variable.

However, the optimum in Fig. 3 is valid for the mean outcome of the random load and is therefore not meaningful from a RBDO standpoint. Instead, consider the reliability at a specific cost threshold, c_o, as a function of the design variable. Fig. 4 shows the variation in the reliability index with the design variable, for three different cost thresholds. The solid line in Fig. 4 shows the variation of the reliability index associated with the threshold $300,000; that is, close to the expected cost in Fig. 2. The non-solid lines show the variation of the reliability index associated with the higher cost thresholds $400,000 and $500,000; that is, in the right tail of the cost distribution in Fig. 2.

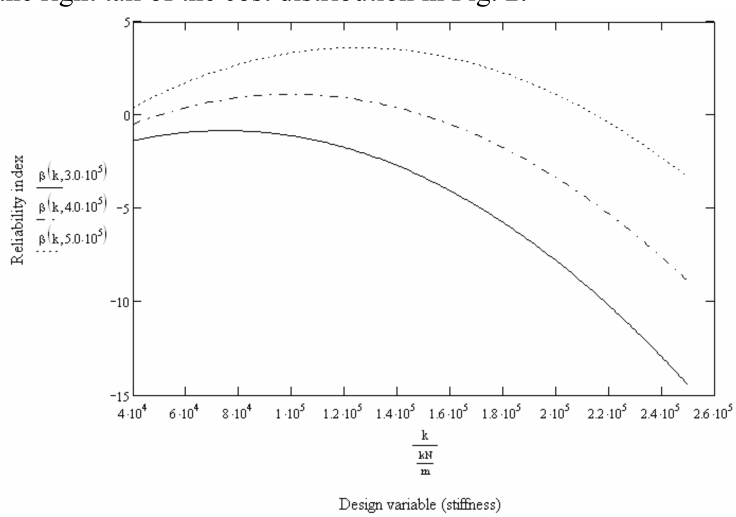

Figure 4: Reliability index associated with three different total cost thresholds; $300,000; $400,000; and $500,000, plotted as function of the design variable (stiffness).

From Fig. 4 it is inferred that the optimal value of the design variable increases with increasing cost threshold. That is, the maximum reliability index for the total cost limit-state function moves to the right when a higher total cost threshold is considered. The total cost threshold $300,000 yields 75,000kN/m as the optimum stiffness; the total cost threshold $400,000 yields 100,000kN/m as the optimum stiffness; and the total cost threshold $500,000 yields 125,000kN/m as the optimum stiffness. In other words, design optimization in the region of potential devastating losses leads to a more conservative design. Hence, risk averseness on the part of the decision maker becomes part of the design. This is akin to the well-known decision analysis concept of employing a utility function with built-in risk averseness (Benjamin and Cornell 1970) to incorporate the averseness against sustaining high losses.

The proposed RBDO strategy can be formulated in two ways. Instead of optimizing the total cost reliability at a selected cost threshold, as is demonstrated in Fig. 4, one may perform the optimization at a selected reliability. To illustrate the idea, Fig. 5 shows the results of the "inverse" reliability analyses. Specifically, the solid line shows the variation in the total cost threshold, c_o, as the design variable is varied, or a fixed probability. The same tendency as above is observed; when the reliability is increased, effectively going further out in the tail of the cost distribution, the optimal design becomes more conservative. Fig. 5 shows that the optimum design at β=1.0 is 98,000kN/m, while the optimum design at β=4.0 is 128,000kN/m. Hence, risk averseness becomes part of the design when higher reliabilities thresholds in the unified reliability analysis are considered.

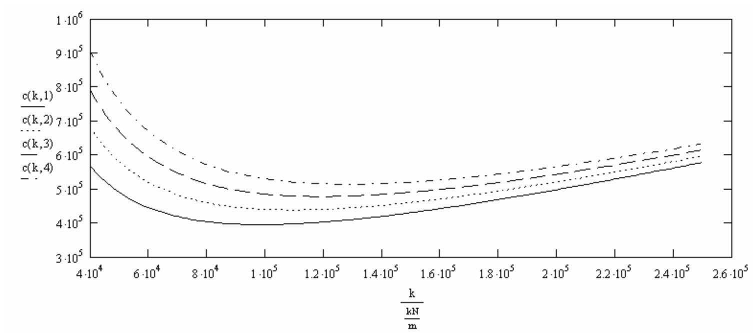

Figure 5: Total cost associated with different levels of reliability; β=1; β=2; β=3; and β=4, plotted as function of the design variable (stiffness).

An interesting observation is now made by means of Fig. 6. This figure shows the probability distribution for the total cost for the three different optimum stiffness values identified in Fig. 4. Notably, the PDF curves that are associated with an optimal design at a higher cost threshold (non-solid lines) yields less probability for a high loss but may yield a higher expected (mean) total cost. This is comforting because the non-solid lines in Fig. 6 results from RBDO in the tail of the probability distribution from the unified reliability analysis, hence it should be expected that it is the "expectation" of such high losses that is minimized.

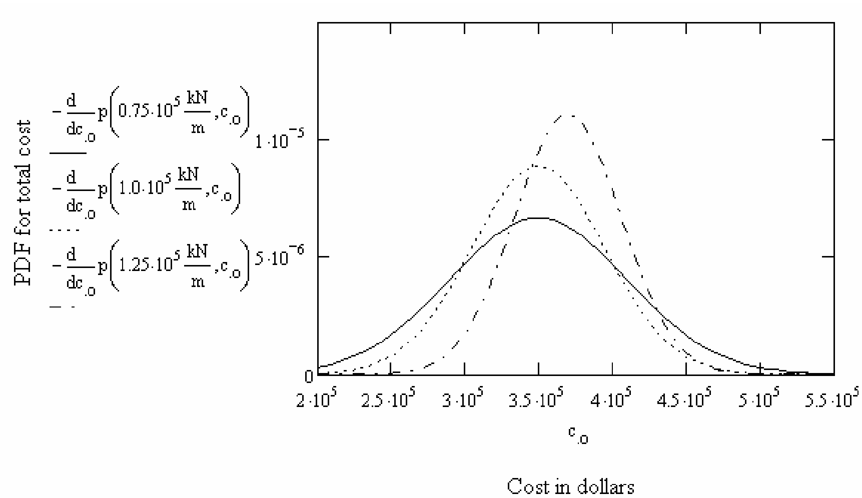

Figure 6: PDF for the total cost for different values of the design variable; associated with different optimum values from Fig. 5.

The concepts explained for the exceedingly simple example above are now generalized to full-scale problems. Relative to the presentation of Figs. 4 and 5, two problems formulations are possible. Fist, the RBDO problem may be cast as the maximization of the total cost reliability at a specific cost threshold:

$$\max\{\beta(\mathbf{d}) \mid c_o = \tilde{c}_o\} \quad \Leftrightarrow \quad \min\{p(\mathbf{d}) \mid c_o = \tilde{c}_o\} \tag{8}$$

where the implicit dependence of the reliability index on the design variables is shown, and $\tilde{c}_o$ is the selected cost threshold. Heuristically, this is akin to taking a point in the tail of the solid-line PDF in Fig. 6 and pushing it downwards. This problem formulation is founded on the discussion of the results shown in Fig. 4. It is also noted that concepts of inverse reliability analysis (Der Kiureghian *et al.* 1994; Li and Foschi 1998) are applicable to solve this problem. Conversely, the problem may be cast as the minimization of the cost threshold in the limit-state function in Eq. (5) for a fixed reliability index equal to the value $\tilde{\beta}$:

$$\min\{c_o \mid \beta = \tilde{\beta}\} \tag{9}$$

Also this problem formulation serves the purpose of reducing the probability content in the tail of the total cost distribution; the region that may be of most serious concern to the decision maker. Upcoming publications demonstrate the implementation and application of this methodology for full-scale building examples in the seismically active region surrounding Vancouver, BC.

Conclusions

The presented unified reliability formulation is suggested as an alternative to evaluate performance-probabilities in earthquake engineering. The methodology places focus on the availability of probabilistic models to predict ground motion, structural response, damage, and loss. Advantages from traditional structural reliability analysis are kept; including accuracy in the assessment of small probabilities and availability of parameter importance measures. In this paper, the unified reliability analysis is employed to facilitate a novel reliability-based design optimization strategy. The design optimization is carried out in the tail of the total cost distribution; that is, at the damage levels that often are of most pressing concern to the stakeholders.

References

Benjamin, J.R., and Cornell, C.A. (1970). Probability, statistics, and decision for civil engineers. McGraw-Hill, New York.

Cornell, C.A. and Krawinkler, H. (2000). Progress and challenges in seismic performance assessment. PEER Center News. http://peer.berkeley.edu/news/2000spring/index.html.

Der Kiureghian, A., Zhang, Y., and Li, C.C. (1994). Inverse reliability problem. Journal of Engineering Mechanics, ASCE 120:1154-9.

Ditlevsen, O. and Madsen, H.O. (1996). Structural reliability methods. John Wiley & Sons.

Enevoldsen, I., and Sorensen, J. (1994). Reliability-based optimization in structural engineering. Structural Safety, 15(3), 169–196.

Gardoni, P., Der Kiureghian, A. and Mosalam K.M. (2002). Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations. Journal of Engineering Mechanics, 128(10), 1024-1038.

Gasser, M. and Schueller, G. (1998). Some basic principles in reliability-based optimization (RBO) of structures and mechanical components. Stochastic programming methods and technical applications, K. Marti and P. Kall (Eds.), Lecture Notes in Economics and Mathematical Systems 458, Springer-Verlag, Berlin, Germany.

Haimes, Y.Y. (1998), "Risk modeling, assessment and management," John Wiley & Sons.

Haukaas, T., Der Kiureghian, A. (2005). Parameter sensitivity and importance measures in nonlinear finite element reliability analysis. ASCE Journal of Engineering Mechanics, 131(10), pp. 1013-1026.

Haukaas T. and Der Kiureghian, A. (2004). Finite element reliability and sensitivity methods for performance-based engineering. Report No. PEER 2003/14, Pacific Earthquake Engineering Research Center, The University of California, Berkeley, CA.

Kirjner-Neto, C., Polak, E., and Der Kiureghian, A. (1998). An outer approximations approach to reliability-based optimal design of structures. Journal of Optimization Theory and Application, 98(1), 1–17.

Koduru, S.D. and Haukaas, T. (2007a). Global seismic reliability analysis of structures with FORM. To be published in the proceedings of the ICASP10, Tokyo, Japan.

Koduru, S.D. and Haukaas, T. (2007b). Seismic reliability anlaysis with probabilistic models for ground motion and structure. To be published in the proceedings of COMPDYN'07; ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, Crete, Greece.

Kuschel, N. and Rackwitz, R. (2000). A new approach for structural optimization of series system. Proceedings of the ICASP8, R.E. Melchers and M.G. Stewart (Eds.), Sydney, Australia.

Li, H., and Foschi, R.O. (1998). An inverse reliability method and its application. Structural Safety. 20 (3) 257-270.

Liang, H., Haukaas, T. and Royset, J.O. (2007). Reliability-Based Optimal Design Software for Earthquake Engineering Applications. Canadian Journal of Civil Engineering, in press.

Mackie, K.R. and Stojadinovic, B. (2006). Fourway: Graphical tool for performance-based earthquake engineering. ASCE Journal of Structural Engineering, 132(8), pp. 1274-1283.

Madsen, H. and Friis Hansen, P. (1992). A comparison of some algorithms for reliability-based structural optimization and sensitivity analysis. Reliability and Optimization of Structural Systems, Proceedings IFIP WG 7.5, R. Rackwitz and P. Thoft-Christensen (Eds.), Springer-Verlag, Berlin, Germany.

Moehle, J., Stojadinovic, B., Der Kiureghian, A. and Yang, T.Y. (2005). An application of PEER performance-based earthquake engineering methodology. Research Digest No. 2005-1, Pacific Earthquake Engineering Research Center, The University of California, Berkeley, CA.

Royset, J.O., Der Kiureghian, A., and Polak, E. (2006). Optimal design with probabilistic objective and constraints. Journal of Engineering Mechanics, 132(1), 107-118.

Yang, T.Y. (2006). Performance evaluation of innovative steel braced frames. Ph.D. Dissertation, The University of California, Berkeley, CA.

Zhu, L., Elwood, K.J. and Haukaas, T. (2007). Classification and seismic safety evaluation of existing reinforced concrete columns. ASCE Journal of Structural Engineering, in press.