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Acceptance Criteria for Deteriorating Structural Systems

Daniel Straub & Armen Der Kiureghian
Department of Civil and Environmental Engineering, University of California, Berkeley
straub@ce.berkeley.edu, adk@ce.berkeley.edu

Abstract

The paper presents a systematic approach to determining acceptance criteria for deteriorating elements in structural systems. Special emphasis is placed on systems with a large degree of redundancy with respect to deterioration failures of individual elements. The effect of statistical dependence among deterioration failures in the system is investigated and it is found that the effect is critical for redundant structural systems. Indicators used in practice for describing the structural importance of individual deteriorating elements fail to account for this dependence and it is proposed to overcome this shortcoming by establishing acceptance criteria from idealized systems. The resulting model is directly applicable in practice.

1. Introduction

Owners of structural systems are confronted with the problem of determining whether their structure and their inspection/maintenance/repair policy can be deemed acceptable with regard to potential deterioration failures. While this applies equally to new-built and to existing structures, the problem is particularly relevant for the latter, for which the cost of increasing the reliability is generally much higher. To date, satisfactory methods for demonstrating acceptability of the risks associated with deterioration in structural systems exist only for special cases. This paper aims at filling this gap by providing a practical yet consistent method to determine reliability-based acceptance criteria for deteriorating elements in general structural systems.

Existing codes typically specify design criteria and safety factors for individual structural elements. This applies for failures caused by static or dynamic overloading of the structural elements (as described by ultimate limit states) as well as for deterioration failures (e.g., described by fatigue limit states). However, deterioration failures exhibit some fundamental differences as compared to overload failures, which make it necessary to explicitly account for the system characteristics. When structural systems collapse because of overloading, all elements involved in the critical failure mode, including cascading failures, normally fail during the same load event. For this reason, the safety margins of the individual elements exhibit a strong statistical dependence and the system reliability approximately equals the reliability of the individual elements if these have equal reliability. Failures of structural elements caused by deterioration, on the other hand, are likely to occur at different times. These events have lower statistical dependence and the system reliability, therefore, substantially differs from the reliability of individual elements. For this reason, the acceptability of deterioration failures must be assessed as a function of structural redundancy. In addition, deterioration can be detected before failure occurs, but deterioration failures can also remain undetected. The inspection and repair policy, therefore, influences the acceptability of deterioration failures. These aspects are partly reflected in some codes, e.g., Eurocode 3 (1992) or NORSOK (1998), where safety factors for fatigue limit states are specified as a function of the consequences of element failure (structural importance) and as a function of the possibility to inspect an element.

In the past, reliability-based acceptance criteria for deterioration limit states have been considered mainly for structures subject to fatigue, in particular fixed offshore structures, e.g., in Kirkemo (1990), Moan and Vardal (2001), Straub and Faber (2005). Thereby, the acceptance criteria were determined as a function of the structural importance of the considered element. This structural importance was assessed based on a simplified approach by comparing the overall capacity of the intact structural system with the capacity of the structural system with the element removed. As will be shown in this paper, this approach is only suitable for elements with a high reliability and if the statistical dependence among deterioration failures is low, because it neglects the joint occurrence of more than one deterioration failure. These conditions are not generally fulfilled. In this paper, an approach is presented that allows determining reliability acceptance criteria for any deteriorating element in structural systems. It accounts for the structural importance of the element, but also for the statistical dependence among deterioration failures throughout the structure and the influence of the inspection and repair policy. The approach takes basis on target reliabilities specified for the entire system, e.g., as suggested in the Probabilistic Model Code of the Joint Committee on Structural Safety, JCSS (2006).

2. Target reliability indices for the structural system

In the Probabilistic Model Code of the Joint Committee on Structural Safety, JCSS (2006), target reliabilities indices β_T for ultimate limit states are specified as a function of the consequences of component failure and as a function of the relative cost of a safety measure. This differentiation reflects the fact that the target reliability indices are based on an optimization of expected life-cycle costs, see Rackwitz (2000). The values are presented in Table 1.

Table 1. Tentative target reliability indices β^T related to a one year reference period and ultimate limit states, provided in JCSS (2006).

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
Large	β^T=3.1 ($p_F{}^T \approx 10^{-3}$)	β^T=3.3 ($p_F{}^T \approx 5 \cdot 10^{-3}$)	β^T=3.7 ($p_F{}^T \approx 10^{-4}$)
Normal	β^T=3.7 ($p_F{}^T \approx 10^{-4}$)	β^T=4.2 ($p_F{}^T \approx 10^{-5}$)	β^T=4.4 ($p_F{}^T \approx 5 \cdot 10^{-6}$)
Small	β^T=4.2 ($p_F{}^T \approx 10^{-5}$)	β^T=4.4 ($p_F{}^T \approx 5 \cdot 10^{-6}$)	β^T=4.7 ($p_F{}^T \approx 10^{-6}$)

According to JCSS (2006), "the values given [in Table 1] relate to the structural system or in approximation to the dominant failure mode." In the absence of owner-specified reliability targets, these values can be considered as the target reliability indices associated with system failure caused by deterioration. Mitigation measures against deterioration typically are expensive, in particular for existing structures, and the target reliability index will be as given in the upper two lines of Table 1. Note that the required reliability indices against overload failure of the structural system can be different if the cost of improving structural capacity is higher or lower than the cost of improving deterioration resistance.

3. Definition and classification of the structural system and elements

3.1. System definition

The structural system is the physical entity that is directly associated with the consequences of deterioration failures. As an example, consider fatigue of welded connections in the primary structure of a bridge. The main consequences of this deterioration process are related to the failure of the entire bridge, which, therefore, is the relevant system to be considered. For certain types of deterioration, multiple consequences can be associated with different systems: Consider a reinforced concrete (RC) bridge deck subject to corrosion of the reinforcement. Here, consequences are due to spalling of the concrete, the relevant system being the lower side of the concrete deck, and due to failure of the entire structure, in which case the total bridge is the system. For this example, acceptance criteria for the structural elements must be determined with respect to both systems and the more stringent one must be applied. The consequence event, the system failure associated with deterioration, is denoted by C_F. This event is defined as the intersection of the event of any deterioration failure in the system, F, and the event of system failure, C: $C_F = \{C \cap F\} = \{C \cap (F_1 \cup F_2 \cup \ldots \cup F_N)\}$. The probability of this event must fulfill

$$\Pr\left(C_F\right) \leq \Phi\left(-\beta_{C_F}^T\right) \tag{1}$$

where $\beta_{C_F}^T$ is the overall target reliability index (Table 1) and $\Phi(\,)$ is the standard Normal cumulative probability function (cdf).

This paper is concerned solely with system failure events C_F that are caused by or related to deterioration in the system. It is assumed that the structural system is designed such that it provides sufficient reliability in an undamaged condition (i.e., without deterioration). Deterioration is then considered as an additional load case and the methods presented in the following aim at ensuring that the system is sufficiently resilient against this additional load case.

3.2. Definition of structural elements (description of deterioration failure)

Deterioration is modeled on the level of structural elements. Such elements can be, e.g., structural members, welded joints, area segments of a continuous surface. The definition of elements depends on the type of deterioration. In the following, it is assumed that deterioration can be represented by a set of limit state functions g, which distinguish the states of failure, F_i, and survival, $\overline{F_i}$, of each element *i*. An example of such a limit state function is:

$$g\left(t\right) = \Delta - At^B \tag{2}$$

where t is the time since installation of the element, Δ is the damage limit and A and B are parameters describing the deterioration process. For B=1, this corresponds to most applied corrosion models as well as to the Miner fatigue model with stresses following a stationary process; for B=0.5, the model is representative of diffusion-controlled deterioration, and for B=2 the model represents deterioration due to sulfate attack. Failure occurs when $g(t) \leq 0$. The elements must be defined such that the representation by a two-state variable is reasonable. As an example, consider the RC bridge example. For the case where spalling of concrete is the relevant system failure event, the limit state function must relate to area segments of the lower surface of the concrete slab, which are the elements of the system. For the case where failure of the entire bridge is the relevant system failure, the capacity of the element will depend on the entire cross-section and the limit state must, therefore, relate to the entire slab, which then is considered as the structural element. In the following, all elements i are fully represented by their deterioration failure events F_i (and their complements $\overline{F_i}$).

Deterioration in an element will occur gradually with time and representing the capacity of an element by a two-state variable can be a strong simplification. In reality, the capacity of a structural system may be significantly reduced even before any of its elements have failed. This effect depends on the nature of the deterioration process, in particular on the loss of element capacity as a function of time. If the loss of capacity is increasing exponentially with time, then the modeling of deterioration by a two-state variable may be considered as a reasonable engineering assumption. This applies, e.g., for deterioration processes that are controlled by a protection system (including corrosion of the reinforcement in concrete), for fatigue and other highly localized deterioration mechanisms. For deterioration processes that lead to a reduction of capacity approximately as a linear function of time, e.g., uniform corrosion on unprotected steel surfaces, application of the presented model requires additional considerations. In these cases, some deterioration is typically accepted (e.g., allowable corrosion losses) and the limit state functions for element failure then have the accepted deterioration limit as a failure criterion.

3.3. Classification of structural elements

A system for classifying structural elements is proposed. The classification is based on the effect of element failure on the structural system (the structural importance of the elements). The four proposed classes are summarized in Table 2.

Table 2. The proposed scheme for classifying structural elements

Name	**Description**	**Redundancy (Structural importance)**	**Examples**
Immediately critical elements	Failure of the element will cause immediate collapse of the structural system with a high probability.	None (determining)	Elements of a minimal structure subjected mainly to dead and live loads; elements of a statically determinate structure; support of a facade element; containment.
Critical elements with delayed failure	Failure of the element is likely to cause collapse of the structural system once an extreme live load (e.g., environmental/ accidental load) occurs.	Little (critical)	Elements of a minimal structure or primary structural elements of a structure subjected to environmental loads; containment under time-variant conditions (pressure etc.).
Redundant structural elements	Failure of a single element has little bearing on the system capacity, but failure of a group of elements can cause system failure.	Large (minor importance)	Most elements of typical structural systems; joint in a ship hull or redundant offshore structure, reinforcement in concrete slab, foundation pile.
Serviceability elements (limit states)	Failures has no bearing on the structural capacity and consequences are limited to reduced serviceability.	Fully (no importance)	Spalling of concrete elements (when no physical damage is caused by the spalled pieces); non-critical deformations.

The first class represents elements whose failure leads to system failure immediately. Examples of such systems are single line pipelines and statically determinate structures. In this case, the structural system is a series system. If deterioration failures of the elements in such systems are statistically independent, then one has $\Pr(C_F) = 1 - \prod_1^N [1 - \Pr(F_i)]$. When determining target reliability indices, all N elements are considered equal and the target reliability index $\beta_{F_i}^T$ for all elements in a series system is thus:

$$\beta_{F_i}^T = -\Phi^{-1}\left\{1 - \Phi\left(\beta_{C_F}^T\right)^{\frac{1}{N}}\right\} \tag{3}$$

However, in reality deterioration failures will not occur independently and Eq. (3) is conservative. If systems have a large number of elements, this must be considered. Computation of the probability of failure of a series system with equi-correlated elements is straightforward, e.g., Thoft-Christensen and Murotsu (1986), and the target reliability index can be computed accordingly.

For the fourth class of elements, the serviceability elements, determination of reliability indices is straightforward. Because these elements are not safety-critical, the owner (the decision maker) is free to choose the target reliability. In JCSS (2006) values for the target reliability indices for serviceability limit states are proposed, but ideally the target reliability criteria should be determined from an economical optimization, or, alternatively, the design should be optimized directly without the need for target reliability indices.

The second and the third class of elements in Table 1 are most common in structural systems. For these elements, the target reliability indices must be determined based on a system model that accounts for the structural importance of the elements. In addition, for these elements it is of importance whether a failure of the element can be detected and repaired before it leads to a failure of the system. The reference period for which target reliabilities are to be determined becomes a function of the time a failure can remain undetected in the system. The rest of the paper is concerned with these types of elements, but the first category (no redundancy with respect to element deterioration failure) can be considered as a special case of this more general situation.

4. Reference time and detectability

As previously noted, deterioration failures, which do not lead to immediate system failure and that are not detected and repaired will persist and may lead to failure of the structure years after they occur. For this reason, the reference period to be applied when assessing the acceptability of deterioration failures must be in accordance with the inspection/repair intervals, unlike for ultimate limit states where the reference period is one year (as in Table 1). Let T_D denote the time period between inspections plus the time elapsing between detection of a failed element and repair thereof. It is assumed that a deterioration failure is certain to be detected upon inspection. For elements where failure can be detected without inspection, the in-between inspection time can be set to zero, if it is ensured that all failures are reported to the organizational unit responsible for repair. Thus, T_D represents the detectability of deterioration failures. The probability that an element i is in a failed state F_i at a given time t is then conditional on no-failure at time $t-T_D$, because if it had failed before $t-T_D$, it would have been repaired. In the following, the performance of a repaired element is assumed to be identical to the performance of an element that has not failed. Thus, the probability of failure of an element at time t given a specific inspection/repair policy (represented by T_D) is

$$\Pr\left[F_i \text{ in } (0,t)\middle|T_D\right] = \frac{\Pr\left[F_i \text{ in } (0,t)\right] - \Pr\left[F_i \text{ in } (0,t-T_D)\right]}{1-\Pr\left[F_i \text{ in } (0,t-T_D)\right]} \tag{4}$$

For the special case where T_D is equal to one year, the above corresponds to the annual failure rate. In the following, let $\beta_{F_i}(t,T_D)$ denote the reliability index of an element at time t for a given inspection/repair policy, i.e., $\beta_{F_i}(t,T_D) = -\Phi^{-1}\{\Pr[F_i \text{ in } (0,t)|T_D]\}$. In order to demonstrate compliance with given acceptance criteria, $\beta_{F_i}(t,T_D)$ must be compared with the target reliability index $\beta_{F_i}^T$. $\beta_{F_i}(t,T_D)$ increases with decreasing T_D, reflecting the beneficial effect of having short inspection/repair intervals. Note that these intervals relate only to detection of already failed elements. It accounts for the fact that the expected consequences of a failure are lower if it is detected fast. Inspections that aim at detecting deterioration before the element fails have an additional effect, namely, they reduce the probability of failure of the element, see, e.g., Straub and Faber (2006).

5. Statistical dependence among deterioration failure events

The events of deterioration failure in a structural system are generally correlated, due to common uncertain influencing factors, such as common environmental conditions and common material characteristics. In most cases, this statistical dependence must be estimated using engineering judgment. It is often possible to get an idea on the order of magnitude of the statistical dependence by explicitly considering the common influencing factors. Alternatively, it may be possible to infer the statistical dependence by comparing within batch variability to in-between batch variability, if such data is available. In Vrouwenvelder (2004) one such study is reported, where the correlation coefficient between the fatigue crack growth parameter C of two welded joints of the considered type in the same structure is estimated as 0.85.

Statistical dependence among deterioration failures can be expressed through the correlation coefficient among the safety margins of the deterioration limit states. As an example, consider the limit state given in Equation (2). This limit state can be reformulated to

$$g(t) = \ln\Delta - \ln A - B\ln t \tag{5}$$

Because the limit state function only distinguishes between $g > 0$ and $g \leq 0$, this formulation is equivalent to Equation (2). If both Δ and A are modeled by a Lognormal distribution and B is modeled by a Normal distribution, the reliability index at time t without any inspection ($T_D \geq t$) becomes:

$$\beta_{F_i}(t) = \frac{\lambda_\Delta - \lambda_A - \mu_B \ln t}{\sqrt{\zeta_\Delta^2 + \zeta_A^2 + (\sigma_B \ln t)^2}} \tag{6}$$

with λ_Δ, ζ_Δ, λ_A and ζ_A being the parameters of the Lognormal distributions of Δ and A, respectively. Exemplarily, statistical dependence is introduced by a correlation coefficient between ln(A) for any pair of elements and between ln(Δ) for any pair of elements, denoted by $\rho_{\ln A}$ and $\rho_{\ln\Delta}$, while B is assumed to be statistically independent from element to element. The correlation coefficient between the safety margins then is

$$\rho_M(t) = \frac{\rho_{\ln\Delta}\zeta_\Delta^2 + \rho_{\ln A}\zeta_A^2}{\zeta_\Delta^2 + \zeta_A^2 + (\sigma_B \ln t)^2} \tag{7}$$

When considering inspections, i.e., for the conditional case as represented by Equation (4), safety margins are no longer Normal distributed and ρ_M is not suitable for describing the statistical dependence. Instead, the correlation coefficient between the binary variables describing the state of the individual elements (failed/survived) can be computed, denoted by $\rho_F(t)$. For the case without inspections, one can easily derive $\rho_F(t)$ as a function of $\rho_M(t)$:

$$\rho_F(t) = \frac{\Phi_2\left[-\beta_{F_i}, -\beta_{F_i}, \rho_M(t)\right] - \Phi^2\left[-\beta_{F_i}(t)\right]}{\Phi\left[-\beta_{F_i}(t)\right] - \Phi^2\left[-\beta_{F_i}(t)\right]} \tag{8}$$

where $\Phi_2[\,]$ is the standard but correlated bi-Normal cdf. For the case including inspections, $\rho_F(t, T_D)$ can be computed from Monte Carlo simulation and an equivalent $\rho_M(t, T_D)$ is then obtained according to Equation (8).

5.1. Numerical illustration

To investigate the effect of detectability on the statistical dependence among deterioration failures, the above correlation coefficients are computed for the parameter values given in

Table 2. Parameters of the random variables in the example.

Parameter	Mean	Std. dev.	Correl.	Distrib.
Δ	1.0	0.3	0.3	LN
A	0.02	0.006	0.5	LN
B	1.0	0.05	0.0	N

(this corresponds to $\lambda_\Delta = -0.0431$, $\lambda_A = -3.95$, $\zeta_\Delta = 0.294$, $\zeta_A = 0.294$, $\rho_{\ln\Delta} = 0.31$, $\rho_{\ln A} = 0.51$).

Figure 1 presents the resulting reliability index of an element as a function of time for two inspection/repair policies: T_D=1yr and $T_D = \infty$, the latter corresponding to the case where a defect cannot be detected or repaired. Figure 2 summarizes the resulting correlation coefficients for the two cases.

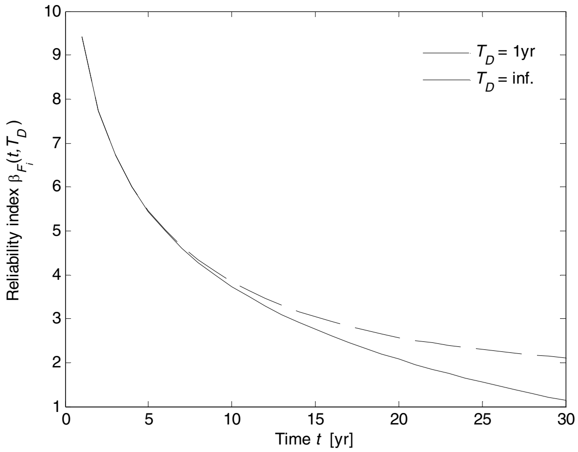

Figure 1. Reliability indices as a function of time for two different levels of detectability.

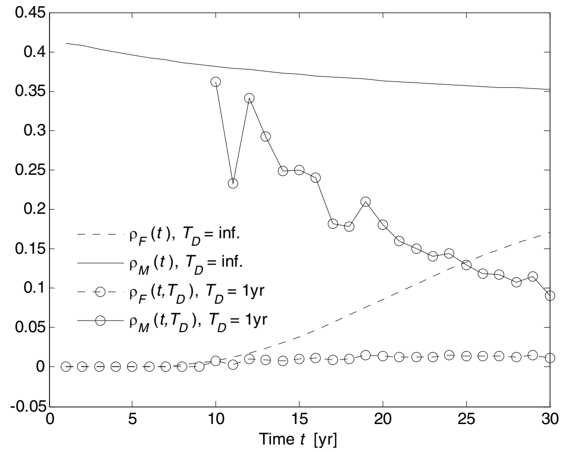

Figure 2. Correlation coefficients as a function of time for two different levels of detectability. (For T_D=1yr, the correlation coefficients are obtained from MC simulation).

The effect of detectability on the reliability of the element, as illustrated by Figure 1 and discussed earlier, has been realized in the past. Additionally, as observed in Figure 2, detectability has a strong influence on the statistical dependence. This effect has not been demonstrated previously. Because statistical dependence among elements has a significant influence on the reliability in redundant systems, as will be shown later, this is highly relevant and should be included in the assessment of target reliability indices for the structural elements. If the detection/repair period T_D is relatively short, then the correlation coefficient should be reduced, in accordance with the numerical example shown in Figure 2.

6. System modeling

The system is represented by two variables. The first variable has two states and describes whether the system is in a failed state, C, or in a functioning (safe) state, $\overline{C}$. The second variable, which is denoted by ψ, describes the state of the system with regard to deterioration. Considering that there are N elements (i.e., potential locations of deterioration failures) in the system, each of which is modeled by a two-state variable (F_i or $\overline{F_i}$), there are 2^N potential system configurations with respect to element deterioration failures, i.e., the sample space of ψ is comprised of 2^N mutually exclusive states. As an example, if elements 2 and 3 have failed through deterioration, and all others are in an intact state, then $\psi = \{\overline{F}_1, F_2, F_3, \overline{F_4}, \ldots, \overline{F_N}\}$. The consequences of deterioration failures are associated with the failure of the system. Therefore, to assess these, it is required to determine the effect of ψ on $\Pr(C)$. Theoretically, it is possible to compute the conditional probabilities $\Pr(C|\psi)$ by performing structural reliability analyses (SRA) of the structure with the elements damaged according to ψ (i.e., all elements that are failed are removed in the structural model employed in the SRA). However, this would require a large number of analyses, which is unfeasible for general structural systems.

6.1. Accounting for structural importance of the elements

In the past (e.g., Kirkemo 1990, Moan and Vardal 2001, Straub and Faber 2005), the influence of individual deterioration failures F_i has been appraised by computing the probability of system failure with element i removed and all other elements intact: $\Pr(C|\psi = \{\overline{F}_1, \ldots \overline{F_{i-1}}, F_i, \overline{F_{i-1}}, \ldots, \overline{F_N}\})$. As an example, in Straub and Faber (2005), the following indicator is proposed for determining acceptance criteria for the individual elements:

$$SEI_i = \Pr\left(C \middle| \psi = \left\{\overline{F}_1, \ldots \overline{F_{i-1}}, F_i, \overline{F_{i-1}}, \ldots, \overline{F_N}\right\}\right) - \Pr\left(C \middle| \psi = \overline{F}\right) \tag{9}$$

wherein $\overline{F}$ denotes no deterioration failures in the entire system and *SEI* denotes Single-Element Importance measure. The SEI_i is the difference in failure probability of the system with all elements intact (not deteriorated) and the system where element *i* has failed due to deterioration. Acceptance criteria for all elements are obtained as

$$\beta_{F_i}^T = -\Phi^{-1}\left[\frac{1}{N}\frac{\Phi\left(-\beta_{C_F}^T\right)}{SEI_i}\right] \tag{10}$$

This approach requires only one additional SRA per element, which makes it practically feasible. Its disadvantage, as discussed in Straub and Faber (2005), is that it neglects the combined effect of two or more deterioration failures. The approximation is reasonable when the individual structural elements have large deterioration reliability and when the statistical dependence among deterioration failures is low. In this case, the probability of two or more deterioration failures becomes so low that all system failure events involving more than one deterioration failure may be neglected. Unfortunately, these conditions are not fulfilled for most elements in real structures. To investigate the approximation, in the following an idealized system, for which all the above quantities are easily computable, is investigated.

6.2. The SEI for a Daniels system

Consider the structural system illustrated in Figure 3. It corresponds to a Daniels system (Daniels, 1945) for the case of ideally elastic-brittle behavior (case a). All vertical elements are considered exchangeable, i.e., they are described by the same probabilistic model. In Gollwitzer and Rackwitz (1990) the characteristics of this system are examined for a variety of element behaviors. The system is well suited to represent the load-sharing present in structural systems, with the two cases (a and b) as extremes of the true material behavior. Note that the distinction between brittle and ductile element failures relates to failure modes due to overloading of the structure. In the case of deterioration failures it is considered that the elements have no remaining capacity. While elements in real structural systems are normally not exchangeable, the above system is still well suited when examining target reliabilities for structural elements, if the number of elements in the idealized system, N, is not mistaken for the real number of elements in the structural system. Instead, N should be interpreted as an equivalent number of elements. The determination of N for realistic structural systems is discussed later.

For the idealized system, computation of the *SEI* according to Equation (9) is straightforward. It is

$$\Pr\left(C \middle| \psi = \left\{\overline{F}_1, \ldots \overline{F_{i-1}}, F_i, \overline{F_{i-1}}, \ldots, \overline{F_N}\right\}\right) = \Pr\left(C \middle| N_F = 1\right) \tag{11}$$

$$\Pr\left(C \middle| \overline{F}\right) = \Pr\left(C \middle| N_F = 0\right) \tag{12}$$

where N_F is the number of failed elements.

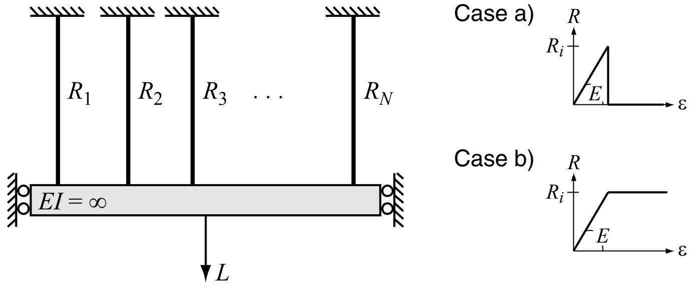

Figure 3. The idealized structural system. Case a) represents brittle element behavior and corresponds to the original Daniels system, case b) represents fully ductile element behavior.

To evaluate Equations (11) and (12), $\Pr(C|N_F = j)$ is required. For given probability distributions of the element capacities R_i and the load L, this is readily obtained for the above system. For case a), it is calculated by

$$\Pr\left(C\middle|N_F = j\right) = \int_L \Pr\left(C\middle|l, n = N - j\right) f_L(l)\, dl \tag{13}$$

where the probability of system failure for given load l and number of elements n, $\Pr(C|l,n)$, is computed according to the solution provided in Daniels (1945) as a function of R_i. For case b), it is calculated as

$$\Pr\left(C\middle|N_F = j\right) = \Pr\left(L - \sum_{i=1}^{N-j} R_i\right) \tag{14}$$

This can be solved using structural component reliability analysis.

The total probability of the system failure associated with deterioration is given as:

$$\Pr\left(C_F\right) = \Pr\left(C \cap F\right) = \sum_{j=1}^{N} \Pr\left(C\middle|N_F = j\right) \Pr\left(N_F = j\right) \tag{15}$$

where $\Pr(N_F = j)$ is a function of the marginal reliability of the individual elements considering deterioration, β_{F_i}, together with the statistical dependence among the elements. Here, this statistical dependence is represented by ρ_M, the correlation coefficient between the deterioration safety margins of any pair of the elements. The probability of j failures among N components then becomes:

$$\begin{aligned} \Pr\left(N_F = j\right) &= \binom{N}{N-j} \int_{-\infty}^{\infty} \varphi(u) \left[\Phi(\beta')\right]^{N-j} \left[\Phi(-\beta')\right]^{j} du \\ \beta' &= \frac{\beta_{F_i} - u\sqrt{\rho_M}}{\sqrt{1-\rho_M}} \end{aligned} \tag{16}$$

where $\varphi(\,)$ is the standard Normal probability density function. This equation is based on a formulation for series systems with equi-correlated elements (Thoft-Christensen and Murotsu, 1986).

6.3. Numerical investigations

To investigate the characteristics of the SEI, a Daniels system with $N = 20$ elements is considered. The load, L, is modeled by a Lognormal distribution with coefficient of variation $\mathrm{COV}[L] = 0.25$ and the capacities of the elements, R_i, are modeled by identical independent Normal distributions with $\mathrm{COV}[R_i] = 0.15$. The mean values of L and R_i are determined such that the system in its undamaged state (without deterioration) has reliability index $\beta_{C_{\bar{F}}} = -\Phi^{-1}[\Pr(C|\bar{F})] = 4.4$ (reference period one year). For this system, $\Pr(C | N_F = j)$ is illustrated in Figure 4. It is observed that the criticality of deterioration failures is almost identical for the two systems if the different material behaviors are considered in the design of the structure against overloading, i.e., if elements that fail in a brittle failure mode are designed with higher safety factors. Therefore, only the system with brittle elements will be considered in the following.

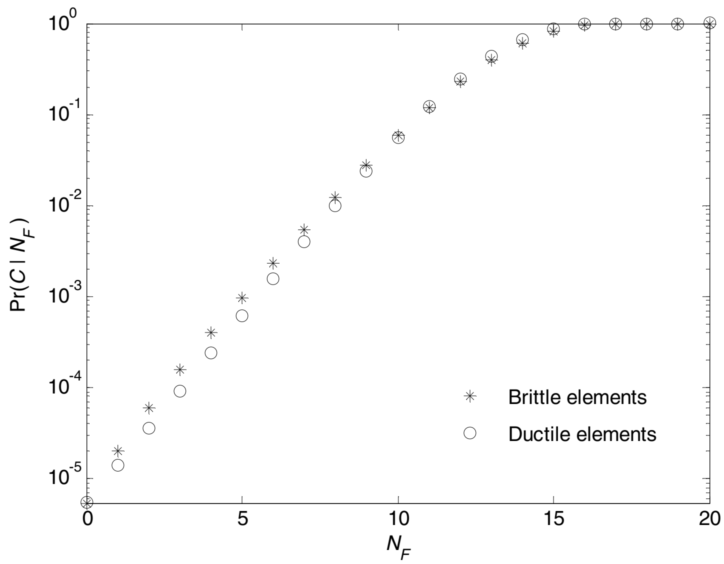

Figure 4. The probability of system failure as a function of the number of failed elements, N_F, for the investigated Daniels system.

With respect to deterioration failure, all elements are assumed to have reliability index $\beta_F = 3.3$ and the pair-wise correlation coefficient among the deterioration safety margins is assumed to be $\rho_M = 0.4$. In Figure 5, the influence of N and ρ_M on the *SEI* and the total probability of system failure associated with deterioration $\Pr(C_F)$ is shown. For the investigated system all elements are exchangeable and the *SEI* is the same for all elements. Because it is considered to apply the *SEI* for determining the target reliability indices, ideally, the shape of the *SEI* should reflect the variations in $\Pr(C_F)$.

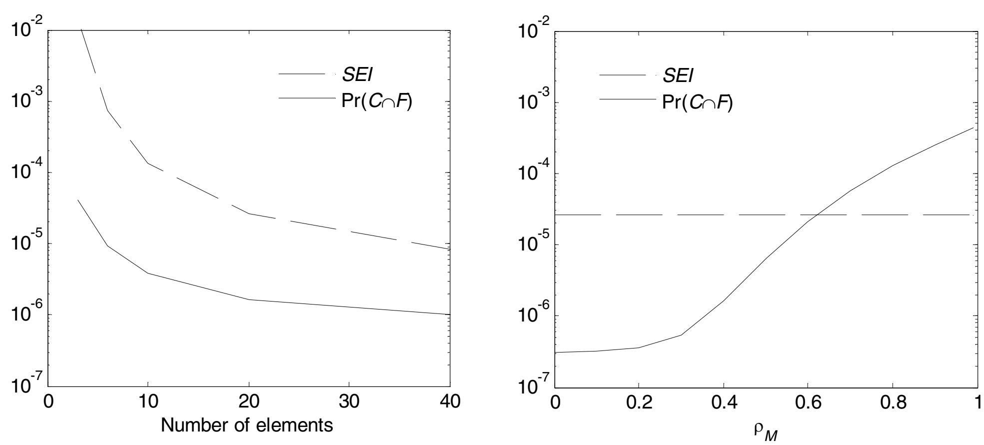

Figure 5. The influence of two parameters on the SEI importance measure; left: the effect of the number of elements N; right: the effect of correlation among deterioration limit states.

Figure 5 shows that the system probability of failure decreases with increasing number of elements *N*, which is reflected by the variation of the *SEI* with *N*. The indicator is thus able to capture the effect of the increase in redundancy when distributing the loads on a larger number of elements. The statistical dependence among deterioration failures in different elements, as represented by ρ_M, has a distinct influence on $\Pr(C_F)$, which, however, is completely neglected by the *SEI*. Considering that $\Pr(C_F)$ varies by three orders of magnitude depending on the value of ρ_M, the example demonstrates that the *SEI* may not be suitable for determining target reliability indices for deterioration in redundant structural systems.

7. Acceptance criteria for deteriorating structural elements in redundant systems

Based on the above observations, a new model for establishing acceptance criteria for deteriorating structural elements in redundant systems is proposed. It is observed that structural systems generally can be represented as a series system of parallel sub-systems (Melchers, 1999). The parallel sub-systems are the failure modes of the structural system. To fully represent a real structure in this format is an intricate process that requires the identification of all possible failure mechanisms (see, e.g., Thoft-Christensen and Murotsu, 1986), and is not feasible for most practical applications. Therefore, it is here proposed to represent the failure modes by an idealized model, namely the Daniels system model presented above, which has the advantage of being easily computable. Because the deterioration target reliability indices $\beta_{F_i}^T$ are determined for each element individually, the system model need not reflect the true structural system behavior accurately. Instead, it is sufficient if the model reflects the effect of β_{F_i} on the reliability of the structural system, β_{C_F}. Therefore, any structural element that is part of a group of load-sharing elements is considered to be part of a Daniels system. For this purpose, for each element *i*, the effect of the remaining load-sharing elements is represented by an equivalent number of statistically equivalent elements. The principle is illustrated in Figure 6 for a simple system consisting only of one failure mode.

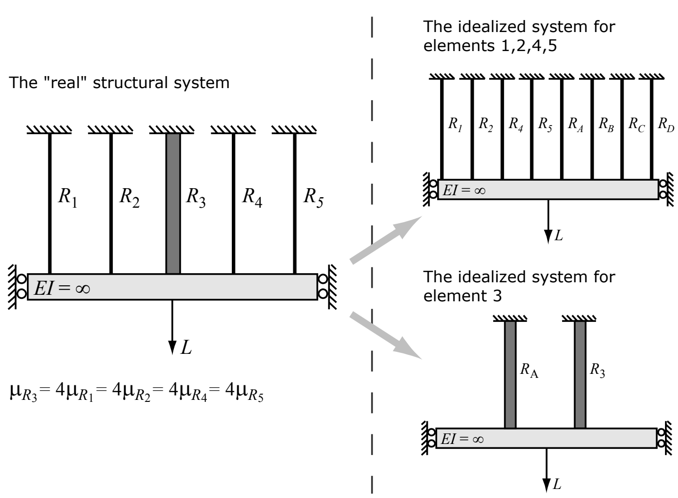

Figure 6. Illustration of the principle of representing a redundant structural system by a number of Daniels systems.

Considering the simple example shown in Figure 6, the error made by approximating the structural system with Daniels systems can be discussed. For element 3, the assumption made is that all remaining elements are grouped into a single element, which corresponds to assuming a full correlation between the deterioration safety margins in all remaining elements. For the elements 1,2,4,5, the assumption is that all other elements have the same statistical dependence, whereas in fact four of the remaining seven element safety margins are fully correlated. In conclusion, for elements with above average importance, the statistical dependence in the remaining system is overestimated, whereas for elements with below average importance, the statistical dependence among the remaining elements is underestimated. Note, however, that the statistical dependence between the considered element and the remaining elements in this failure mode is correctly represented in both cases.

In general, there exist a large number of failure modes of a structural system, which constitute the series system mentioned above. Many of these failure modes, particularly those involving the same structural elements, are strongly correlated. For this reason, it is proposed to consider for each element only the most critical failure mode involving that element. As the failure modes are represented through idealized Daniels systems, no detailed modeling of the failure modes is needed, but rather some simple engineering considerations can be applied to determine the number of failure modes, *K*, and the number of elements in the Daniels system, *N*, for each failure mode. As a conservative approximation it is assumed that these failure modes are independent and equally important. The probability of failure of the k^{th} Daniels system associated with deterioration, $\Pr(C_{F,k})$, is given by Equations (13) - (16) as a function of *N*, the reliability of the elements with respect to deterioration failures, β_{F_i}, the correlation coefficient among deterioration safety margins, ρ_M, and the reliability of the system without deterioration failures, $\beta_{C_{\overline{F}}}$. The latter determines the mean values of the element capacities R_i and the loading L for given distribution families and coefficients of variation of R_i and L. Let the function defining $\Pr(C_{F,k})$ be ϑ, i.e., $\Pr(C_{F,k}) = \vartheta(N, \beta_{C_{\overline{F}}}, \beta_{F_i}, \rho_M)$. With the above assumptions, the total probability of failure of the structural system associated with deterioration is

$$\Pr\left(C_F\right) = 1 - \prod_{k=1}^{K}\left[1 - \Pr\left(C_{F,k}\right)\right] \tag{17}$$

All failure modes are considered equally important, i.e., the acceptable probability of failure is equal for all failure modes *k*. By requiring that $\Pr(C_F) \le \Phi(-\beta_{C_F}^T)$ and by noting that $\Pr(C_F)$ is a small probability value, the following condition is obtained for the Daniels systems:

$$\Pr\left(C_{F,k}\right) \le \frac{\Phi\left(-\beta_{C_F}^T\right)}{K} \tag{18}$$

From this condition, we obtain the acceptance criterion for the reliability index of the individual element for deterioration failures, $\beta_{F_i}^T$, as the value that fulfills the following condition

$$\vartheta(N_i, \beta_{C_{\overline{F}}}, \beta_{F_i}^T, \rho_M) = \frac{\Phi\left(-\beta_{C_F}^T\right)}{K} \tag{19}$$

$\beta_{F_i}^T$ is thus a function of $K, N_i, \rho_M, \beta_{C_{\overline{F}}}, \beta_{C_F}^T$. The subscript *i* in N_i indicates that the number of elements in the Daniels system is a function of the importance of the element *i* and can vary from one element to the next. One possibility of determining N_i is to approximate it by the ratio between the total structural capacity and the contribution of the element to the capacity. This can often be determined from simple engineering considerations. As an example, for a ship structure the global failure mode can be interpreted as a bending failure. The ultimate capacity is approximately proportional to the section modulus at the compression flange (Paik and Mansour, 1996). The contribution of each element (stiffener, plate) to the section modulus is easily computed. Note that in this case the ship structure can fail at any of the midship sections, and each section can fail in a sagging or hogging configuration. The number of these failure modes is *K*.

An alternative way of determining the parameters N_i and *K* is to compute the SEI_i for each element. Following Equation (9), for a given $\beta_{C_{\overline{F}}}$, N_i can be obtained by requiring that

$$\Pr\left(C \middle| N_F = 1\right) = SEI_i + \Phi\left(-\beta_{C_{\overline{F}}}\right) \tag{20}$$

$\Pr(C|N_F = 1)$ depends on the number of elements in the Daniels system, N_i, among other parameters. Therefore, the N_i that corresponds to a particular value of the SEI_i can be obtained from the condition in Equation (20).

The number of failure modes, K, is established through the following simple consideration: As previously stated, for each element only the most relevant failure mode is included in the model (the one that leads to the lowest value of N_i). It then follows that K can be determined from the N_i of all elements in the structure as

$$K = \left\lceil \sum_i \frac{1}{N_i} \right\rceil \tag{21}$$

wherein $\lceil \ \rceil$ is the symbol for rounding to the next higher integer value. This formula gives exact results in the extremes: for a series system with N elements, it is $K = N$ and for a Daniels system it is $K = 1$.

7.1. Numerical investigations

In Figure 7, $\beta_{F_i}^T$ is shown for a number of different combinations of N and ρ_M. Unless otherwise noted, elements are modeled as having brittle material behaviour and the values $K = 1$, $\mathrm{COV}[L] = 0.25$, $\mathrm{COV}[R_i] = 0.15$, $\beta_{C_{\overline{F}}} = 4.4$ and $\beta_{C_F}^T = 3.7$ are used. These values correspond to the case of a structure with large consequences of failure and with normal cost of safety measures against overload failures and large cost of safety measures against deterioration failures, Table 1. The "base case" in Figure 7 shows the results for this case.

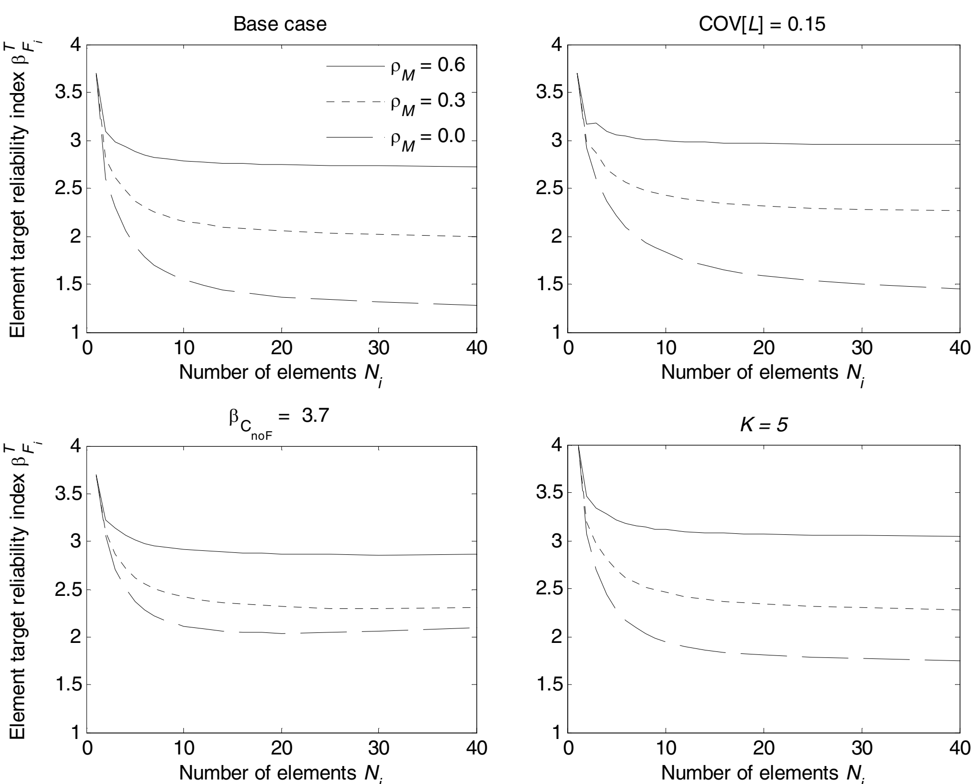

Figure 7. Target reliability indices as a function of the equivalent number of elements and the statistical dependence among deterioration failures for different system configurations.

$1/N_i$ can be interpreted as representing the structural importance of element *i*. From Figure 7 it is observed that for an N_i larger than a relatively small number (5 to 10), the target reliability index is rather insensitive to N_i. This indicates that for structural elements with a low structural importance it is not necessary to exactly determine the level of redundancy of the structure with respect to failure of the element. The results in Figure 7 furthermore point out the relevance of the statistical dependence among deterioration failures. Here, ρ_M is the most relevant parameter for structural elements with a medium to low structural importance. As observed on the upper right of Figure 7, the coefficient of variation of the loading on the structure has a distinct influence on $\beta_{F_i}^T$. When the uncertainty in the loading is reduced relative to the uncertainty in the total structural capacity *R*, then the sensitivity of the reliability of the structure to *R*, α_R, is increased. This leads to an increase in the structural importance of the element and, consequently, to a higher $\beta_{F_i}^T$. On the lower left of Figure 7, the effect of decreasing the reliability of the system without deterioration, $\beta_{C_{\bar{F}}}$, is demonstrated: In order to achieve the same system reliability with respect to deterioration failures, the reliability of the individual elements against deterioration failures must be increased. This points out that under certain circumstances it might be more efficient to increase the reliability of the structural system (e.g., by adding additional structural members) than to increase the reliability against deterioration. When the number of failure modes, K, is increased, the target reliability indices become higher in accordance with Equations (18) and (19), as shown on the lower right of Figure 7.

In accordance with Equation (20), it is possible to relate the SEI_i computed for elements in a real structural system to an idealized Daniels system, by computing the N_i that corresponds to the SEI_i. The target reliability as obtained directly from the SEI_i, Equation (10), and the procedure proposed in this paper can then be compared. This comparison is shown in Figure 8 for the case of $\beta_{C_{\bar{F}}} = 4.4$ and $\beta_{C_F}^T = 3.7$ for a system with $K = 5$. The corresponding total number of elements (required to compute the target reliability index from the SEI_i) is $N_i K$, corresponding to the case where all structural elements in the system are of equal importance.

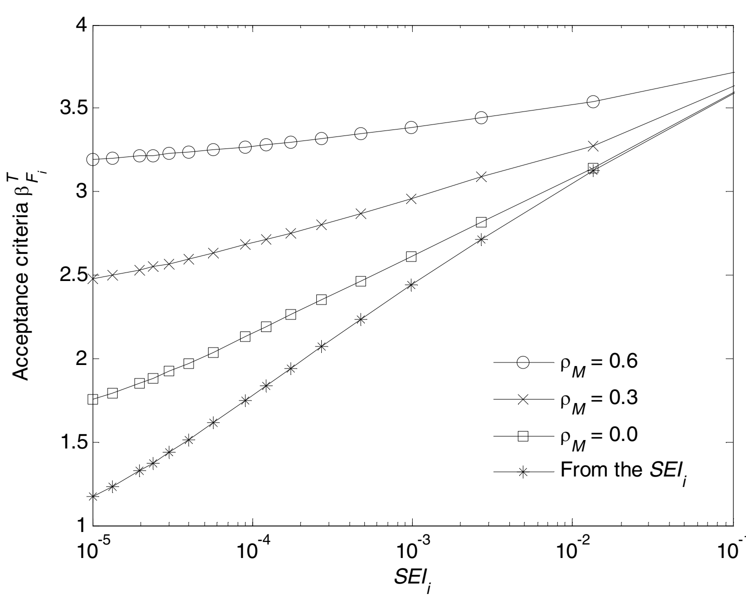

Figure 8. Comparing target reliability indices obtained according to the proposed Daniels system model with those obtained directly from the SEI_i.

Figure 8 clearly demonstrates that neglecting the system effects is non-conservative, in particular for systems with strong statistical dependence among deterioration failures. In this example, the target reliability indices determined through the simple formulation in Equation (10) that does not account for the combined effect of several failures, leads to lower values of $\beta_{F_i}^T$ in all cases, except for $SEI_i = 1$. In this case, the two approaches give the same solution, as system effects become irrelevant (failure of the element automatically leads to failure of the system).

8. Concluding remarks

As illustrated by the numerical examples in this paper, system effects are relevant when determining target reliability indices for redundant structural systems, in particular when deterioration failures in the system elements are statistically dependent. There is strong evidence that such statistical dependence is high for many deterioration mechanisms in structural systems. A full analysis of the system, which includes system reliability assessments for all combinations of deterioration failures, is not feasible for general structures. Therefore, in the past, simple indicators for structural importance of elements have been used to model the effect of an element failure on the integrity of the structure, such as the *SEI* discussed here. However, these indicators fail to account for the effects of multiple deterioration failures and, therefore, for the effect of statistical dependence among deterioration at different elements. For this reason, it has been proposed here to represent the system effects by a number of Daniels systems for the purpose of determining acceptance criteria for individual structural elements. This is an idealization of the true system, which facilitates computation while still capturing the characteristics of structural redundancy (load-sharing among elements) and statistical dependence among deterioration at the elements. Indicators for the structural importance of the system elements, such as the *SEI*, can be used to determine the characteristics of the idealized systems. Additionally, it has been found that for systems with medium to large structural redundancy, the target reliability index for an element is relatively insensitive to its structural importance (represented by the number of elements N_i in the idealized system). For such elements it is thus sufficient to estimate the element structural importance by simple engineering considerations.

For most types of deterioration, statistical dependence among element failures is due to uncertain common influencing factors. However, statistical dependence can also be caused by functional (physical) dependences among structural elements, in particular cascading failures, such as fatigue cracks growing from one element to the next. In most cases, such effects may be considered within the presented model by choosing the statistical dependence among element failures, represented by the correlation coefficient ρ_M, accordingly. However, care is needed when pursuing this approach, because the failure may originate at any of the elements and then propagate to other elements, thus increasing the probability of failure of the individual elements in addition to increasing the statistical dependence among element failure events.

Because statistical dependence among deterioration failures has a negative effect on the reliability of redundant structural systems, it is relevant that the ability to detect and repair deterioration failures may influence this statistical dependence. This influence has been demonstrated here with a numerical example, and the results suggest that, besides the well-known effect of increasing element reliability, inspections also reduce the statistical dependence among element failure events and consequently may justify a reduction of the target reliability indices for the individual structural elements. Through further studies and quantification of this reduction in statistical dependence, a methodology for determining the effect of simple inspections and monitoring systems (such as flooded member detection in offshore structures) may be developed in the future.

The philosophy behind the approach for determining acceptance criteria as presented here is that structural systems are designed such that they provide a given reliability in an undamaged condition (i.e., without deterioration). Deterioration is then considered as an additional load case and it must be ensured that the system is sufficiently resilient against this additional load. The presented system model and the conclusions drawn from application of that model, therefore, apply to any type of loading for which the structure was not explicitly designed. For this reason, the presented model should be further explored in the context of modeling resistance against unforeseen events.

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