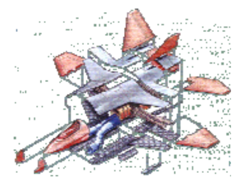
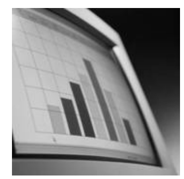
Special workshop on Risk Acceptance and Risk Communication March 26-27, 2007

Risk-quantification of Complex Systems by Matrix-based System Reliability Method



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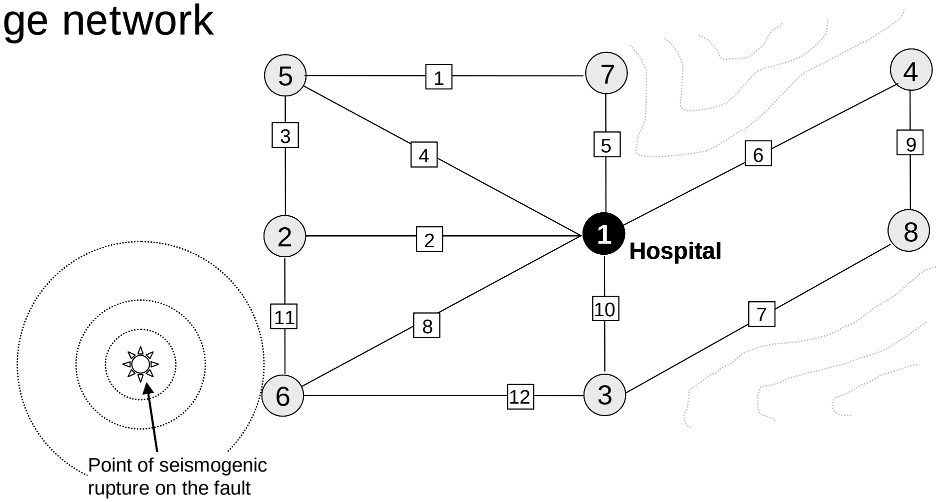
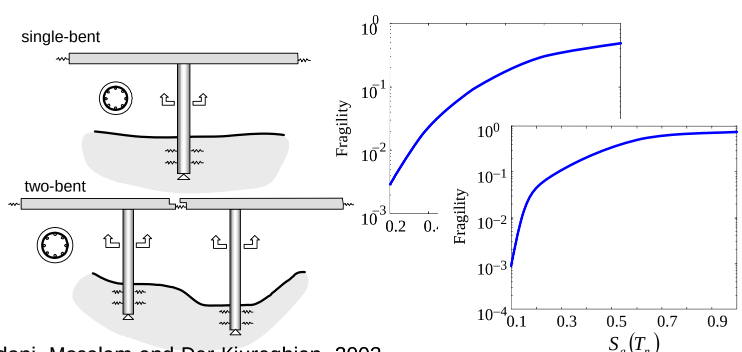
System risk estimates for decision-making

* Bridge network



Component risk estimates

- Component fragilities
- Probability of component failure



Gardoni, Mosalam and Der Kiureghian, 2003

System risks considered for decision-making

- Likelihood of disconnection
- Duration of disconnection
- Number of failed bridges
- Social/economic loss
  (e.g. business disruption)
- Relative importance of components
- Sensitivity of system failure w.r.t. parameters

Contents

- Challenges in system reliability analysis methods
- **Matrix-based system reliability** (MSR) method
  - Matrix-based formulation of system events and probabilities
  - Statistical dependence between components
  - Incomplete information ("LP Bounds" method)
  - Conditional probability & importance measures
- Applications
  - Post-earthquake connectivity of a transportation network
  - Seismic damage of a bridge structure system
  - Progressive failure of a truss structure

Challenges in system reliability

- Complexity of system event description
  - Difficult to identify cut sets or link sets
  - Boolean description ~ lengthy; inconvenient to handle
  - Makes system reliability *analysis* complex as well
- Statistical dependence between components
  - “Environment dependence” or “common source effects”
  - Expensive or infeasible to provide complete information on dependence ~ theoretical bounding formulas
- Incomplete information
  - Not very flexible in incorporating various information
- Statistical inference for decision-making

Existing system reliability methods

- Theoretical bounding formulas (Ditlevsen 1979)

$$P_1 + \sum_{i=2}^{n} \max\left(P_i - \sum_{j=1}^{i-1} P_{ij}, 0 \right) \le P\left(\bigcup_{k=1}^{n} E_k \right) \le P_1 + \sum_{i=2}^{n} \left(P_i - \max_{j<i} P_{ij} \right)$$

- FORM approximation (Hohenbichler and Rackwitz 1983)

$$P(E_{series}) = 1 - \Phi(\beta, \mathbf{R}) \quad P(E_{parallel}) = \Phi(-\beta, \mathbf{R})$$

- Monte Carlo simulations

$$P(E_{\text{system}}) = \int_D f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \cong \frac{\#(\mathbf{x} \in D)}{\#(\mathbf{x})}$$

- LP bounds method (Song and Der Kiureghian 2003)
  → generalized to a **Matrix-based System Reliability** (MSR) method

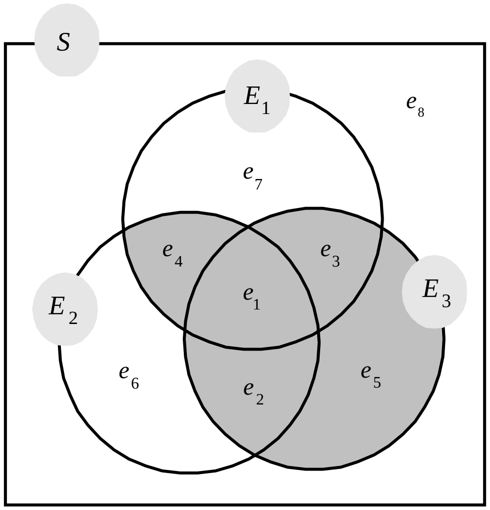
Matrix-based Formulation

- Matrix-based formulation of system failure:

$$P(E_{sys}) = \mathbf{c}^{\mathrm{T}}\mathbf{p}$$

* Example:

$$P(E_1E_2 \cup E_3) = p_1 + p_2 + p_3 + p_4 + p_5$$
$$= [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0] \cdot$$
$$[p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5 \quad p_6 \quad p_7 \quad p_8]^{\mathrm{T}}$$



- **c**: "**event**" vector
  ~ describes the system event of interest

- **p**: "**probability**" vector
  ~ likelihood of component joint failures

Identification of event vector, c

- Matrix-based event operations:

$$\mathbf{c}^{\overline{E}} = \mathbf{1} - \mathbf{c}^{E}$$

$$\mathbf{c}^{E_1 \cdots E_n} = \mathbf{c}^{E_1} .* \mathbf{c}^{E_2} .* \cdots .* \mathbf{c}^{E_n}$$

$$\mathbf{c}^{E_1 \cup \cdots \cup E_n} = \mathbf{1} - (\mathbf{1} - \mathbf{c}^{E_1}) .* (\mathbf{1} - \mathbf{c}^{E_2}) .* \cdots .* (\mathbf{1} - \mathbf{c}^{E_n})$$

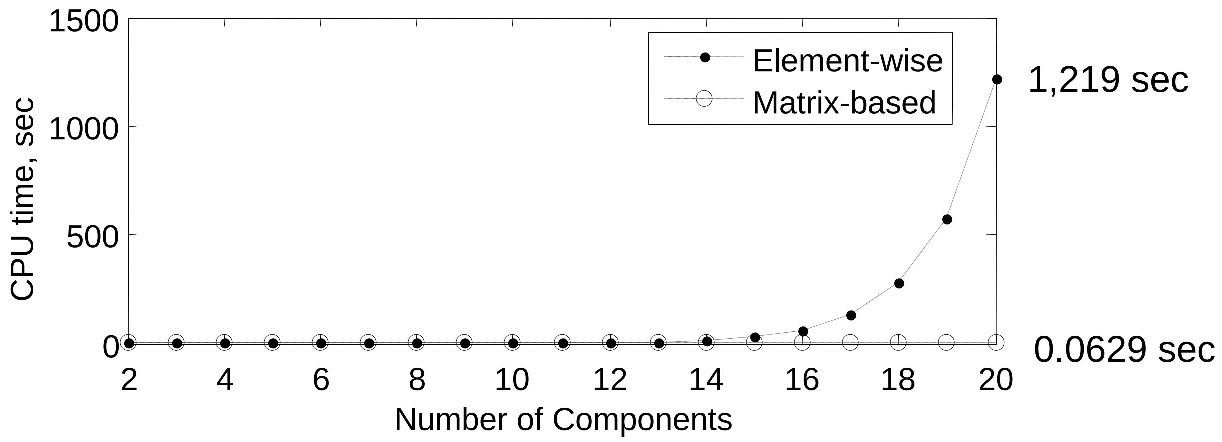
  - Efficient and easy to implement by matrix-based computing languages, e.g. Matlab®, Octave
  - Can construct directly from event vectors of components and other system events
  - Can develop/use problem-specific algorithms to identify event vectors

Computation of probability vector, p

- Iterative matrix-based procedure for statistically independent (s.i.) components

$$\mathbf{p}_{[1]} = \begin{bmatrix} P_1 & 1 - P_1 \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{p}_{[i]} = \begin{bmatrix} \mathbf{p}_{[i-1]} \cdot P_i \\ \mathbf{p}_{[i-1]} \cdot (1 - P_i) \end{bmatrix} \quad \text{for } i = 1, \ldots, n$$



Statistical dependence b/w components

- By total probability theorem,

$$P(E_{sys}) = \int_{\mathbf{x}} P(E_{sys} \mid \mathbf{x}) f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}$$
$$= \int_{\mathbf{x}} \mathbf{c}^{\mathrm{T}} \mathbf{p}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$
$$= \mathbf{c}^{\mathrm{T}} \tilde{\mathbf{p}}$$

  - Utilize **conditional s.i.** of components given an outcome of random variables **X** causing component dependence e.g. Earthquake magnitude for a bridge system
  - Event vector **c** is independent of this consideration ~ no need to construct the probability vector for new system events

"What if not explicitly identified?"

- Example: approximation by Dunnett-Sobel (DS) correlation matrix (1955)

$$Z_i \sim N(\mathbf{0},\mathbf{R}),\ \rho_{ij} = r_i \cdot r_j$$
$$Z_i = \sqrt{1-r_i^2}U_i + r_i X,$$

  - Z_i, $i=1,\ldots,n$ are conditional s.i. given $X=x$
  - Fit the given correlation matrix with a DS correlation matrix with the least square error
  - Can generalize it further for better approximations

$$Z_i \sim N(\mathbf{0},\mathbf{R}), \rho_{ij} = a_i a_j + b_i b_j$$
$$Z_i = \sqrt{1-a_i^2-b_i^2}U_i + a_i X + b_i Y$$

Incomplete information

- LP bounds method (Song and Der Kiureghian 2003)

$$\begin{aligned} &\text{minimize(maximize)} \quad \mathbf{c}^{\mathrm{T}}\mathbf{p} \\ &\text{subject to} \quad \mathbf{A}_1\mathbf{p} = \mathbf{b}_1 \\ &\qquad\qquad\quad \mathbf{A}_2\mathbf{p} \geq \mathbf{b}_2 \\ &\qquad\qquad\quad \mathbf{A}_3\mathbf{p} \leq \mathbf{b}_3 \end{aligned}$$

  - $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$: event vectors for which probabilities or bounds are available
  - $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$: available probabilities or bounds
  - Has been successfully applied to various systems (Song and Der Kiureghian 2003a, 2003b, 2006)

Conditional prob. / importance measure

- Conditional probability Importance Measure (CIM)

$$CIM_i = P(E_i \mid E_{sys}) = \frac{P(E_i E_{sys})}{P(E_{sys})}$$

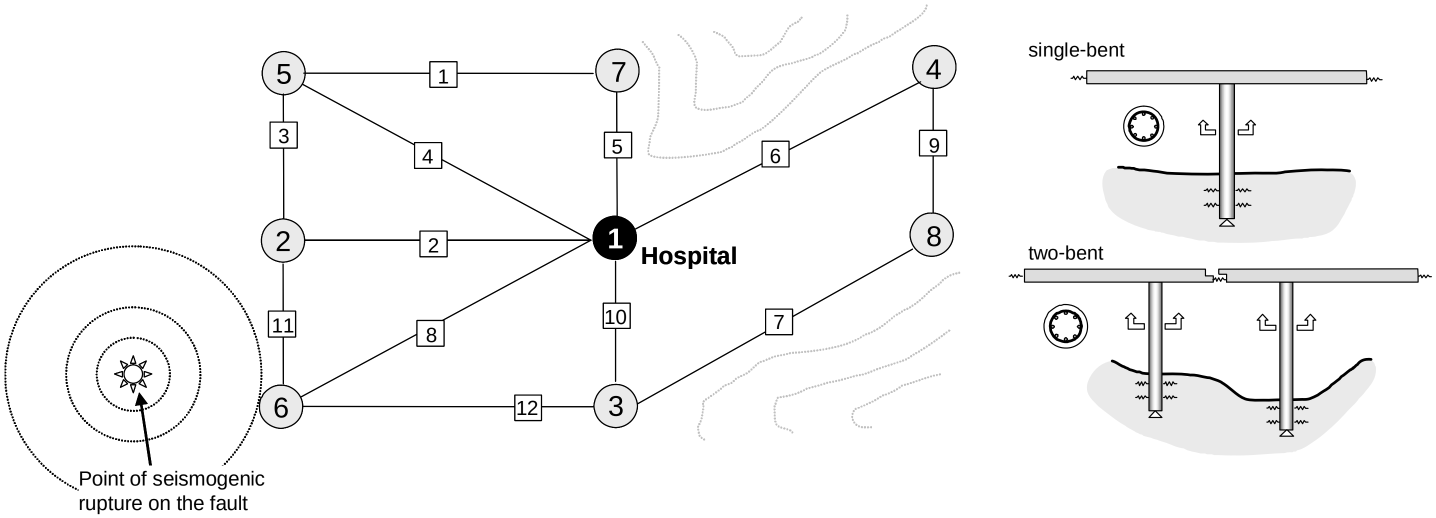
- Fussell-Vesely IM

$$FV_i = \frac{P(\bigcup_{k: C_k \supseteq E_i} C_k)}{P(E_{sys})}$$

  - $P(E_{sys}')/P(E_{sys}) = (\mathbf{c}'^T\mathbf{p}) / (\mathbf{c}^T\mathbf{p})$
  - Once the system reliability is done, only additional task is to find the event vector for a new system event

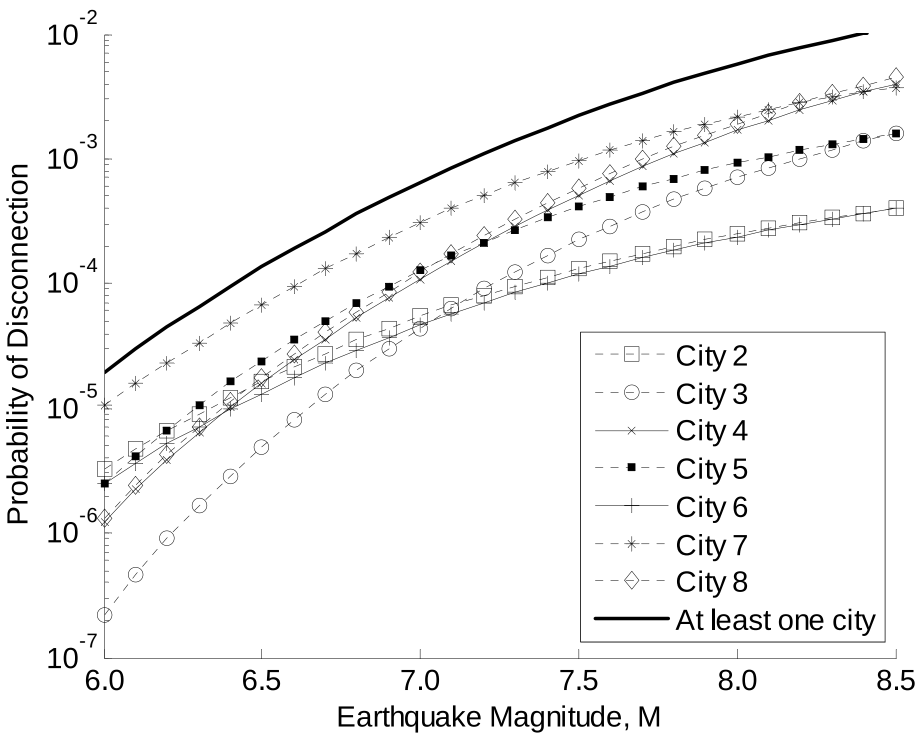
Appl. I: Connectivity of a transportation network

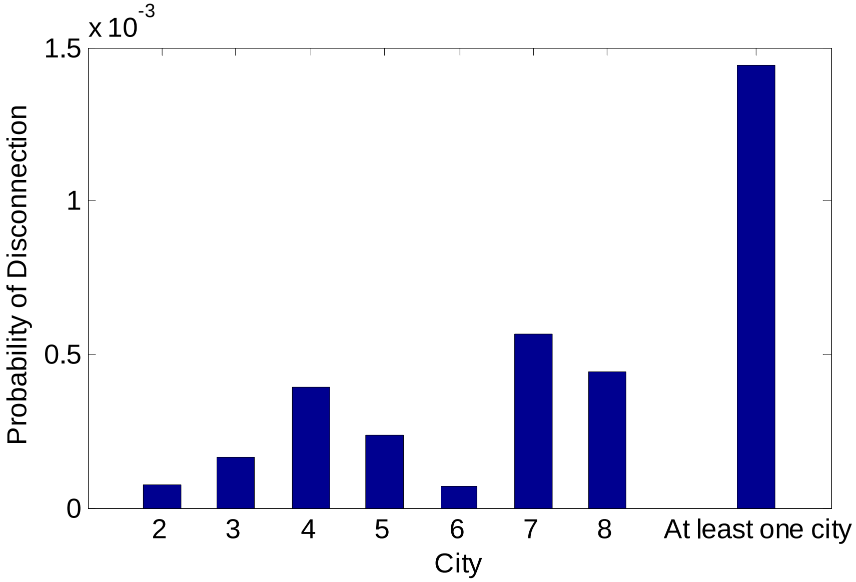
Kang, Song and Gardoni (2007)
~ ICASP10 (July); Reliability Engineering and System Safety (under review)



- Post-earthquake disconnection from the critical facility
- Fragilities for bridges (Gardoni et al. 2003)
- Deterministic attenuation relationship used
- For given magnitude, the bridge component failures are conditional s.i.

Appl. I: Connectivity of a transportation network





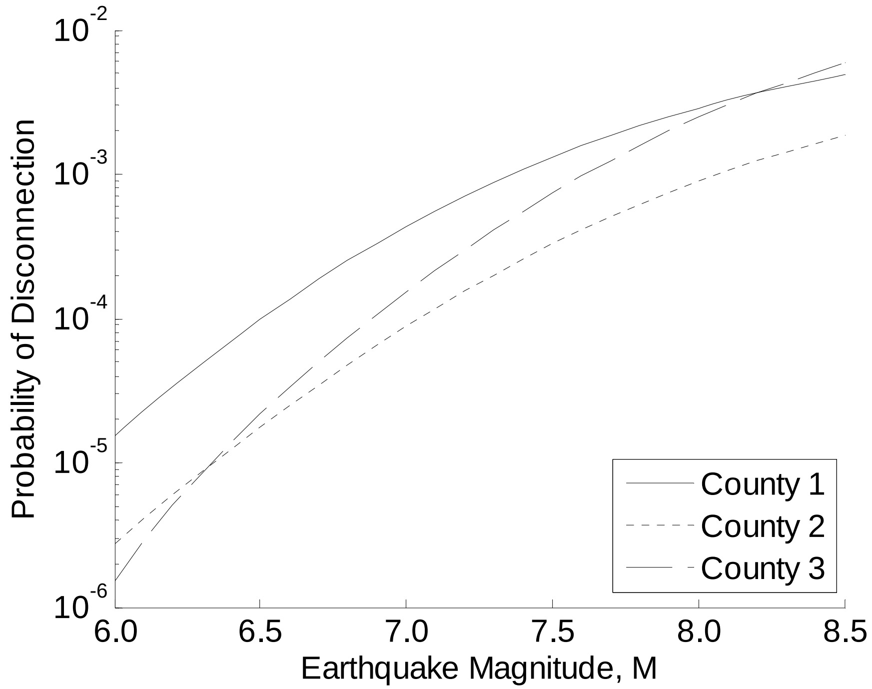
$$P(E_{sys} \mid M = m) = \mathbf{c}^{\mathrm{T}}\mathbf{p}(m)$$

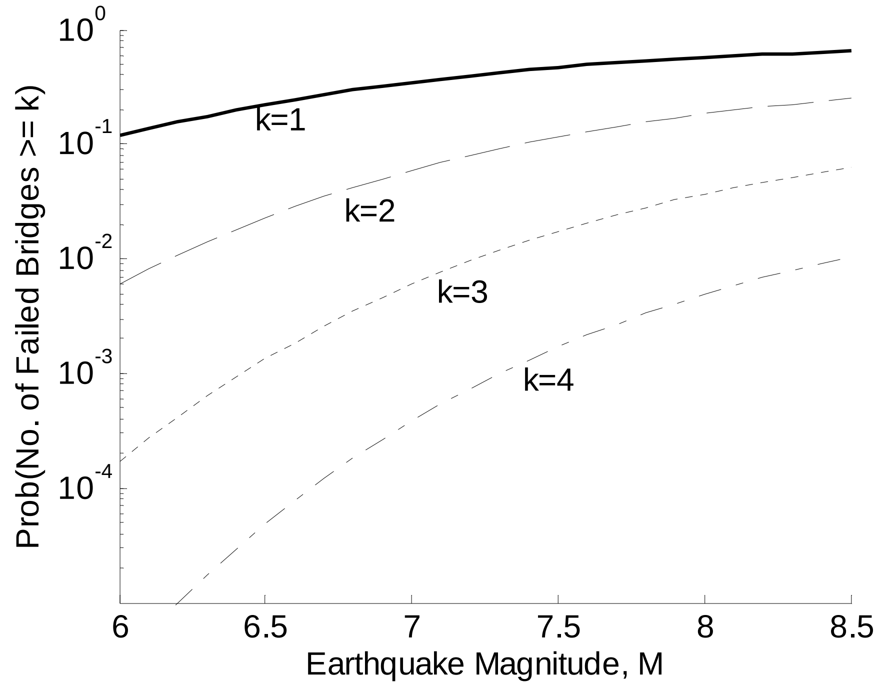
Conditional probability of disconnection of cities

$$P(E_{sys}) = \int_{m_0}^{m_c} \mathbf{c}^{\mathrm{T}}\mathbf{p}(m)\, f_M(m)dm = \mathbf{c}^{\mathrm{T}}\tilde{\mathbf{p}}$$

Probability of disconnection of cities

Appl. I: Connectivity of a transportation network





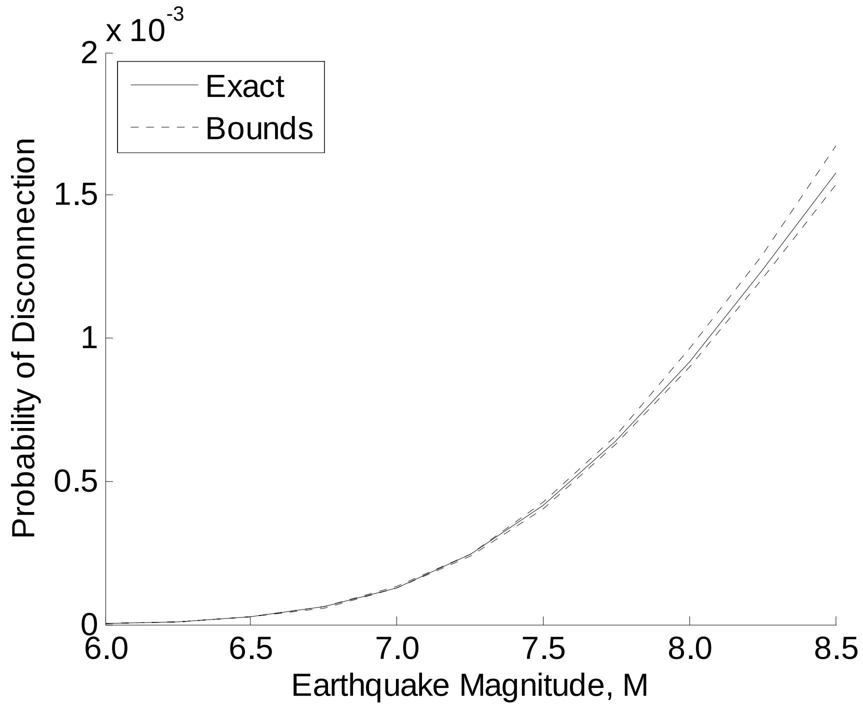
$$P(E'_{sys}) = \mathbf{c}'^{\mathrm{T}}\mathbf{p}(m)$$

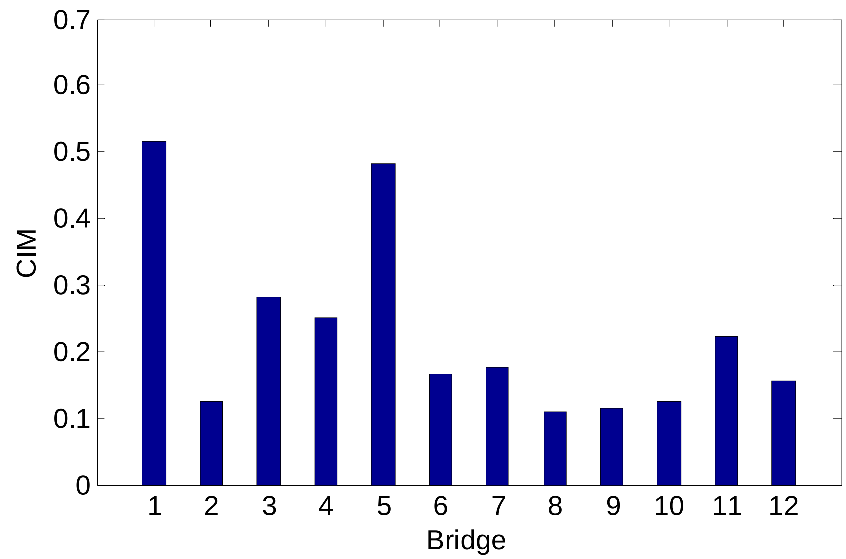
$$P(E'_{sys}) = \mathbf{c}''^{\mathrm{T}}\mathbf{p}(m)$$

Conditional probability of disconnection of counties

Prob (No. of failed bridges ≥ *k*)

Appl. I: Connectivity of a transportation network





$$\min(\max) \quad \mathbf{c}^{\mathrm{T}}\mathbf{p}(m)$$

Bounds on P(City 5 disconnected)

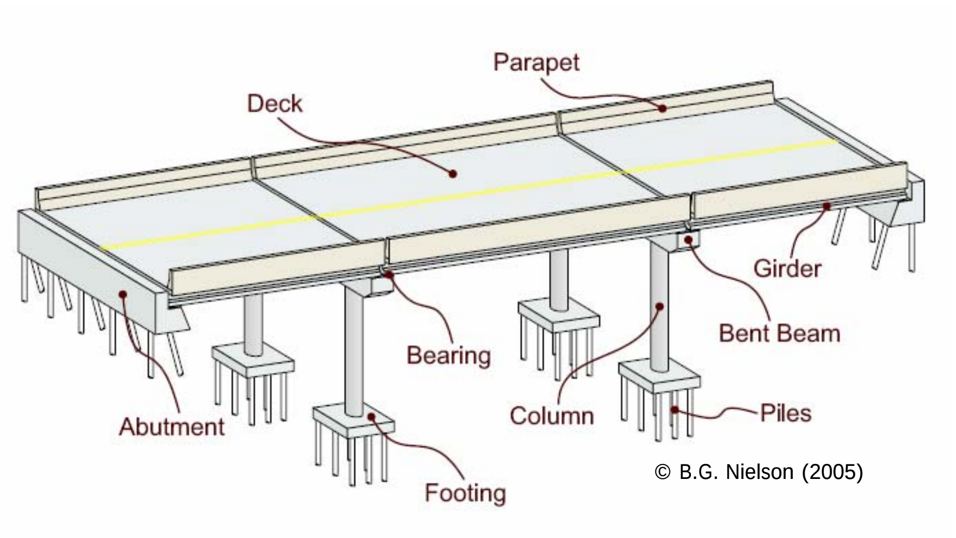
(No information on Bridge 12)

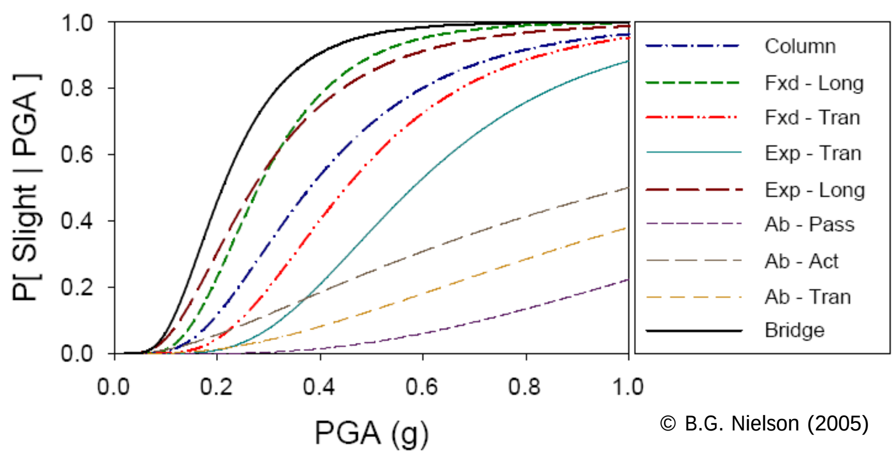
$$P(E_i \mid E_{sys}) = \frac{P(E_i E_{sys})}{P(E_{sys})} = \frac{\mathbf{c}'^{\mathrm{T}}\tilde{\mathbf{p}}}{\mathbf{c}^{\mathrm{T}}\tilde{\mathbf{p}}}$$

Importance measure of components
w.r.t. the likelihood of at least a disconnection

Appl. II: Damage of a bridge structural system

Song and Kang (2007) ~ ASCE EMD conference (June)





- Nielson (2005) developed analytical fragilities of bridge components such as bearings, abutments and columns
- Identified the statistical dependence between demands
- Probability that at least one component fails (series system)
- Performed MCS to account for component dependence

Appl. II: Damage of a bridge structural system

* Safety Factor $F_i = \ln C_i - \ln D_i$

* Fragility

$$P(LS_i \mid IM) = P(F_i \le 0 \mid IM)$$

$$= P\left(Z_i \le -\frac{\mu_{F_i}}{\sigma_{F_i}} \mid IM \right)$$

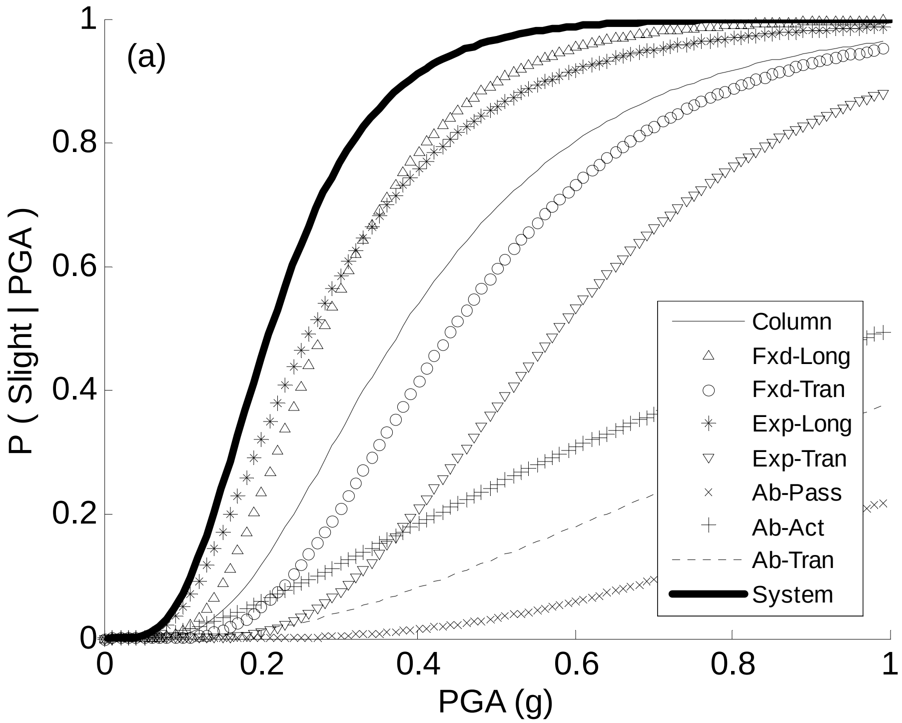
$$= \Phi\left[-\frac{\mu_{F_i}(IM)}{\sigma_{F_i}(IM)} \right]$$

* Correlation

$$\rho_{Z_i Z_j} = \rho_{F_i, F_j} = \frac{(\zeta_{D_i} \cdot \zeta_{D_j})}{(\zeta_{C_i}^2 + \zeta_{D_i}^2)^{1/2} (\zeta_{C_j}^2 + \zeta_{D_j}^2)^{1/2}} \cdot \underline{\rho_{\ln D_i, \ln D_j}}$$

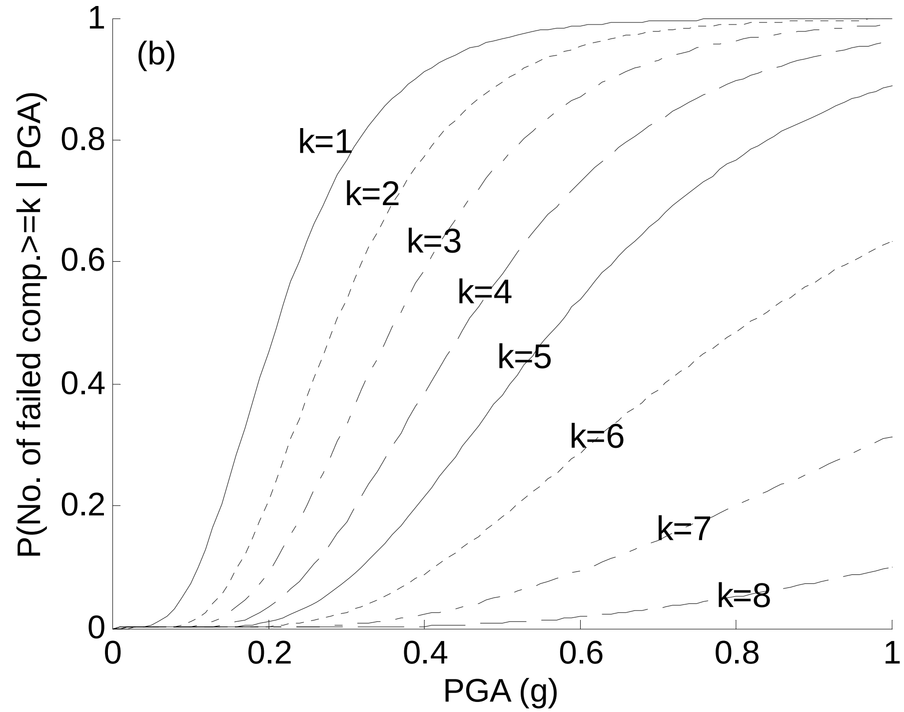
* Fitting by DS-class corr. matrix: average of percentage error ~ 3%

Appl. II: Damage of a bridge structural system



$$P(E_{sys} \mid PGA = pga) = \mathbf{c}^{\mathrm{T}}\mathbf{p}(pga)$$
$$= \int \mathbf{c}^{\mathrm{T}}\mathbf{p}(pga, x)\varphi(x)dx$$

System fragility (at least one)

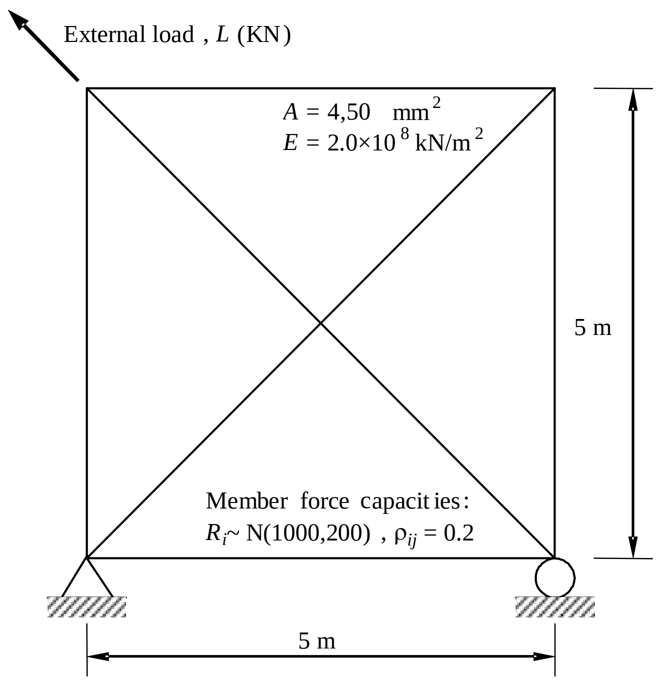


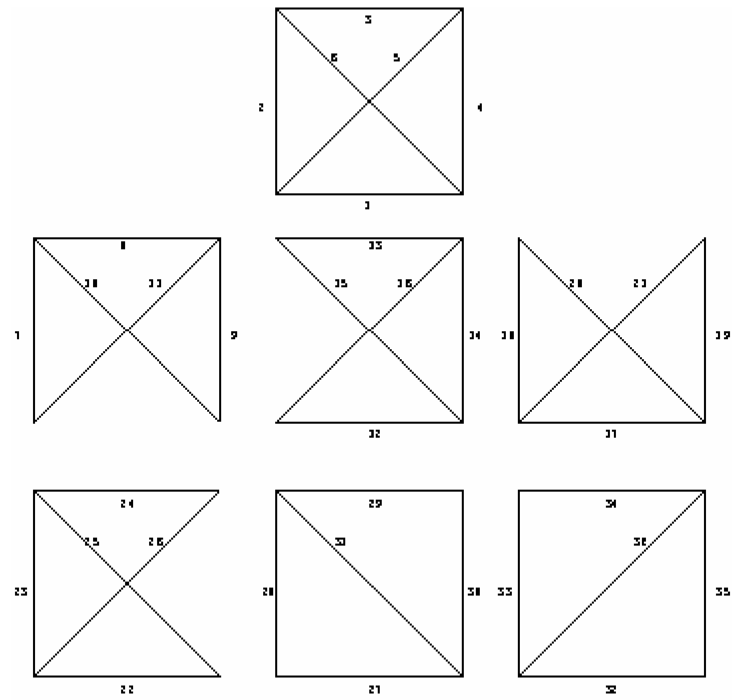
$$P(E_{sys} \mid PGA = pga) = \mathbf{c}'^{\mathrm{T}}\mathbf{p}(pga)$$

P(No. of failed components ≥ k)

Appl. III: Progressive failure of a truss structure

Song and Kang (2007) ~ ASCE EMD conference (June)





$$P(\overline{E}_{sys}) = P[\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6 \bigcup (E_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6)(\overline{E}_7\overline{E}_8\overline{E}_9\overline{E}_{10}\overline{E}_{11})$$

$$\bigcup (\overline{E}_1E_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6)(\overline{E}_{12}\overline{E}_{13}\overline{E}_{14}\overline{E}_{15}\overline{E}_{16}) \bigcup \cdots$$

$$\bigcup (\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5E_6)(\overline{E}_{32}\overline{E}_{33}\overline{E}_{34}\overline{E}_{35}\overline{E}_{36})]$$

Appl. III: Progressive failure of a truss structure

$$P(\overline{E}_{sys}) = P[\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6 \cup (E_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6)(\overline{E}_7\overline{E}_8\overline{E}_9\overline{E}_{10}\overline{E}_{11})$$
$$\cup (\overline{E}_1 E_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6)(\overline{E}_{12}\overline{E}_{13}\overline{E}_{14}\overline{E}_{15}\overline{E}_{16}) \cup \cdots$$
$$\cup (\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5 E_6)(\overline{E}_{32}\overline{E}_{33}\overline{E}_{34}\overline{E}_{35}\overline{E}_{36})]$$

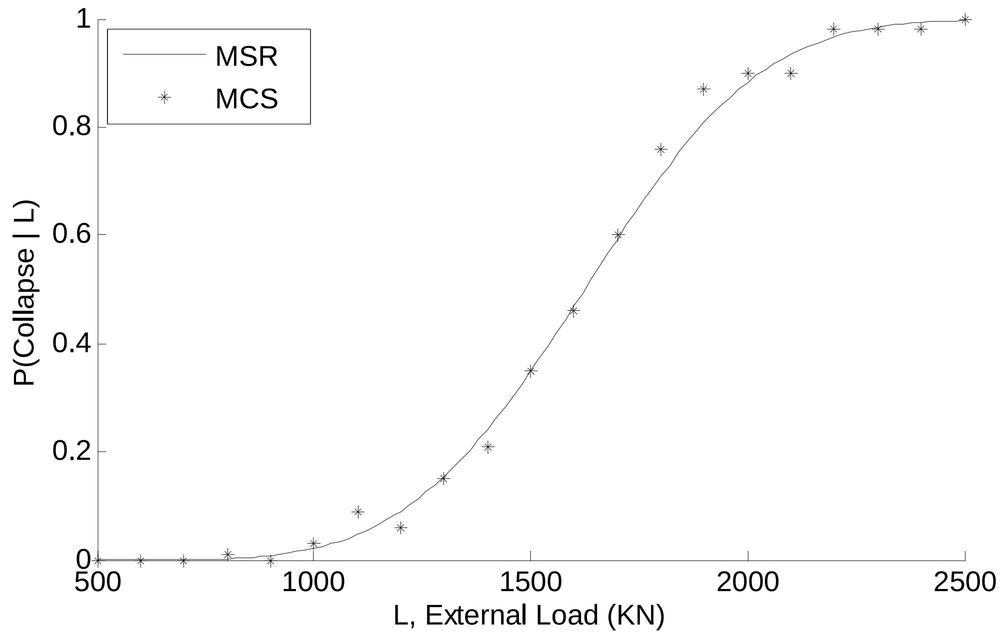
Disjoint link sets (36→11)

$$P(\overline{E}_{sys}) = P(\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6) + P(E_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6\overline{E}_7\overline{E}_8\overline{E}_9\overline{E}_{10}\overline{E}_{11})$$
$$\cdots + P(\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5 E_6\overline{E}_{32}\overline{E}_{33}\overline{E}_{34}\overline{E}_{35}\overline{E}_{36})$$

Perfect correlation

7 systems with 6 components

Appl. III: Progressive failure of a truss structure



- System collapse fragility curve given abnormal load
- Verified through MCS
- Importance of members (components)
- Sensitivity of fragility w.r.t. design parameters

Thank You!