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Risk-quantification of complex systems by matrix-based system reliability method

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Abstract

Many infrastructures are subject to risks that are described as complex "system" events, i.e., logical functions of other "component" events. The complexity of the system event definition, the statistical dependence between the components, and the lack of complete information may prevent accurate and efficient assessment of such system risks. A Matrix-based System Reliability (MSR) method enables us to assess system risks by use of simple matrix calculations. Unlike existing system reliability methods whose complexity highly depends on that of the system event, the MSR method describes any general system event in a simple matrix form and therefore provides a more convenient and uniform framework of handling the system event and estimating its probability. Even in case one has incomplete information on the component probabilities, the matrix-based framework helps obtain the narrowest bounds on the system failure probability by linear programming. This paper presents the MSR method and demonstrates it by numerical examples on the connectivity of a transportation network, the seismic damage of a bridge structure system, and the progressive failure of a truss structure.

1. Introduction

Many structures and lifelines are complex "systems" whose states are described as the Boolean (or logical) functions of "component" events such as the occurrences of structural failure modes and the failures of constituent members. For optimal decision-makings on structural designs, retrofits, repairs, and damage mitigation strategies, it is essential to assess such system risks in an efficient and accurate manner. However, the computation of the probability of such a system event is often costly or infeasible due to the complexity of the system event, the statistical dependence between components, and the lack of complete information. For example, existing system reliability methods such as theoretical bounding formulas (Ditlevsen 1979) and first order reliability method approximations (Hohenbichler and Rackwitz 1983) are applicable to only series and parallel systems with little flexibility in incorporating various types and levels of available information. Song and Der Kiureghian (2003a) proposed a method for computing bounds on the failure probability of any general systems with high flexibility in incorporating available information. The method divides the sample space into mutually exclusive events and describes the system failure probability and the available information by use of vectors and matrices. By solving a linear programming (LP), one can obtain the narrowest possible bounds on the system failure probability for given information.

Song and Kang (2007) generalized this LP bounds method to a Matrix-based System Reliability (MSR) method in order to make use of the matrix-based formulation of system events and probabilities for cases with complete information as well. Unlike existing system reliability methods, the MSR method is uniformly applicable to general systems regardless of their complexity because both the system event and the joint probabilities of components are always described by vectors or matrices regardless of the complexity of the system. Since the matrix representation of a system can be obtained by algebraic manipulations of matrices representing component events or other system events, the MSR method provides a convenient way of identifying/handling the system events as well. Even in case complete component information is not available or affordable, one can still obtain the narrowest bounds on the system probability by solving an LP. This is equivalent to the LP bounds method. This paper presents the MSR method and demonstrates it by three numerical examples. First, the likelihoods of the disconnections in a transportation network are quantified based on the seismic vulnerability of its constituent bridges (Gardoni et al. 2003). We can also estimate probabilities of various system events such as the probability mass function of the number of failed bridges and the conditional probabilities of bridge failures given a disconnection. Second, we use the MSR method for estimating the likelihood of seismic damage of a highway bridge structure. Using the analytical fragilities of various bridge components and the statistical dependence between the seismic demands (Nielson 2005), we estimate the probability that at least one component fails without performing Monte Carlo simulations. Various other system risks are assessed as well. It is demonstrated how the MSR method can deal with common source effect in a structural system. Finally, we consider a statically indeterminate truss structure whose member capacities are uncertain and correlated. The conditional probabilities of the progressive collapse given various levels of an external load are efficiently estimated by the MSR method.

2. Matrix-based system reliability method

2.1. Matrix-based formulation of system reliability

Consider a system whose *i*-th component has s_i distinct states, $i = 1, \ldots, n.$ The sample space can be divided up to $m = \prod_{i=1}^{n} s_i$ mutually exclusive and collectively exhaustive (MECE) events. We name these the "basic" MECE events and denote them by e_j, $j = 1, \ldots, m.$ One can describe any general event as the union of all the basic MECE events that belong to it. Therefore, any general system event can be represented by an "event" vector $\mathbf{c}$ whose *j*-th element is 1 if e_j belongs to the system event and 0 otherwise. Let $p_j = P(e_j),\ j = 1, \ldots, m,$ denote the probability of e_j. Due to the mutual exclusiveness of e_j's, the probability of a system event $P(E_{sys})$ is simply the sum of the probabilities of e_j's that belong to the system event. Therefore, the system failure probability is computed by a simple vector calculation

$$P(E_{sys}) = \sum_{j: e_j \subseteq E_{sys}} p_j = \mathbf{c}^{\mathrm{T}} \mathbf{p} \tag{1}$$

where $\mathbf{p}$ is the "probability" vector that contains p_j's. The formulation in Eq. 1 can be generalized to compute the probabilities of multiple system events under multiple conditions of component failures by a single matrix multiplication, i.e. $P(E_{sys}) = \mathbf{C}^{\mathrm{T}} \mathbf{P}$ where $\mathbf{C}$ and $\mathbf{P}$ are the matrices containing various system vectors and probability vectors, respectively. We name this as a Matrix-based System Reliability (MSR) method (Song and Kang 2007), which has the following merits over existing system reliability methods. First, the complexity of the system reliability computations is not affected by that of the system event definition because the probability of a system event is calculated by a single matrix multiplication regardless of the system definition. Second, the matrix-based formulation allows us to identify/handle the system events conveniently and compute the corresponding probabilities efficiently. Third, even if one has incomplete information on the component failure probabilities and/or their statistical dependence, the matrix-based framework still enables us to obtain the narrowest possible bounds on any general system event (Song and Der Kiureghian 2003a). Fourth, once $P(E_{sys})$ is obtained, one can easily calculate the probabilities of other system events, conditional probabilities and component importance measures without further probability calculations. Finally, the recent developments of matrix-based computer languages and software including MATLAB® and Octave have rendered matrix calculations more efficient and easier to implement. A drawback of the MSR method is that the sizes of vectors and matrices increase exponentially with the number of component events, which may require enormous capacity of computing memory in case of systems with large number of components. However, we can overcome this by transforming a large system problem into multiple smaller problems by the multi-scale approach (Der Kiureghian and Song 2007) or describe the system event by aggregated, non-basic MECE events directly identified by advanced algorithms.

2.2. Identification of event vector $\mathbf{c}$

For small-size systems, one could identify the event vector $\mathbf{c}$ directly. However, this approach may become infeasible or inefficient as the size of the system increases. An important merit of the MSR method is that one can construct the event vector of a system event by simple matrix manipulations of the event vectors of components or other subsystem events. Let $\mathbf{c}^E$ denote the event vector of a generic event E. The event vectors of the complementary event of E, the intersection and the union of events $E_1, E_2, \ldots, E_n$ are respectively obtained by

$$
\begin{aligned}
\mathbf{c}^{\bar{E}} &= \mathbf{1} - \mathbf{c}^{E} \\
\mathbf{c}^{E_1 \cdots E_n} &= \mathbf{c}^{E_1} .* \mathbf{c}^{E_2} .* \cdots .* \mathbf{c}^{E_n} \\
\mathbf{c}^{E_1 \cup \cdots \cup E_n} &= \mathbf{1} - (\mathbf{1} - \mathbf{c}^{E_1}) .* (\mathbf{1} - \mathbf{c}^{E_2}) .* \cdots .* (\mathbf{1} - \mathbf{c}^{E_n})
\end{aligned}
\tag{2}
$$

where $\mathbf{1}$ denotes a vector of 1's; and "**.***" is the MATLAB® operator for element-by-element multiplication. Using a matrix-based language, one can perform these calculations simply by single-line expressions with improved efficiency. In case the system event has not been identified as a Boolean description due to the complexity or a large number of cut sets or link sets, one can use/develop a problem-specific computer algorithm to construct the event vector from the vectors of components or other system events.

2.3. Computation of probability vector $\mathbf{p}$

Let us first consider a system whose component failure probabilities are all available and the component events are statistically independent. In this case, each element of the probability vector is the product of the probabilities of components or their complementary events that include the corresponding basic MECE event. If each element is computed one by one, this can be an expensive numerical task, especially for systems with large number of components. In order to overcome this, we propose to construct the probability vector by the following iterative matrix-based procedure:

$$
\begin{aligned}
\mathbf{p}_{[1]} &= \begin{bmatrix} P_1 & 1 - P_1 \end{bmatrix}^{\mathrm{T}} \\
\mathbf{p}_{[i]} &= \begin{bmatrix} \mathbf{p}_{[i-1]} \cdot P_i \\ \mathbf{p}_{[i-1]} \cdot (1 - P_i) \end{bmatrix} \text{ for } i = 2, 3, \ldots, n
\end{aligned}
\tag{3}
$$

where $\mathbf{p}_{[i]}$ denotes the probability vector for a system with components $\{1, \ldots, i\}$; and P_i denotes the probability of the *i*-th component. This matrix-based procedure can construct the probability vectors much more efficiently than element-wise computations. In a numerical test with MATLAB®, the CPU time to construct the probability vector for a system with 20 components were 1,219.0 sec by the element-wise calculation while it took only 0.0629 sec by Eq. 3 (Kang et al. 2007).

2.4. Statistical dependence between components

When component events have statistical dependence, it may be a daunting task to construct the $\mathbf{p}$ vector because the probabilities of the basic MECE events can not be computed by the products of P_i's and $(1-P_i)$'s. However, in many structural system reliability problems, we can achieve conditional independence between component events given outcomes of a few random variables representing the "environmental dependence" or "common source effects." For example, when system events related to a bridge network are considered, the component (bridge) failures may be considered conditionally independent of the failures of other bridges given a seismic intensity. Let $\mathbf{X}$ and $f_{\mathbf{X}}(\mathbf{x})$ denote a vector of random variables causing the statistical dependence between components and its joint probability density function (PDF), respectively. Then, by the total probability theorem and the MSR formulation in Eq. 1, the system failure probability can be computed as

$$P(E_{sys}) = \int_{\mathbf{x}} P(E_{sys} \mid \mathbf{x}) f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x} = \int_{\mathbf{x}} \mathbf{c}^{\mathrm{T}} \mathbf{p}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \mathbf{c}^{\mathrm{T}} \tilde{\mathbf{p}} \quad (4)$$

where $P(E_{sys} \mid \mathbf{x})$ denotes the conditional failure probability of the system given outcome $\mathbf{X}=\mathbf{x}$; and $\mathbf{p}(\mathbf{x})$ is the vector of the conditional probabilities of the basic MECE events given $\mathbf{X}=\mathbf{x}$. The conditional independence of the components given $\mathbf{X}=\mathbf{x}$ enables us to construct $\mathbf{p}(\mathbf{x})$ efficiently by the iterative matrix manipulations in Eq. 3. The "predictive" probability vector, $\tilde{\mathbf{p}}$ can be obtained by a numerical integration of $\mathbf{p}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x})$ over the space of $\mathbf{x}$. It is noteworthy that even with the consideration of statistical dependence, we need to perform the matrix multiplication only once, i.e., $\mathbf{c}^{\mathrm{T}}$ and $\tilde{\mathbf{p}}$. This approach is not necessarily limited to the cases in which the "common source" random variables are identified explicitly. For example, if the component capacities, demands or safety margins follow a Dunnett-Sobel (DS) class correlation matrix (Dunnett and Sobel 1955), their statistical dependence can be represented by a single random variable. This is demonstrated by the last two numerical examples of this paper.

2.5. Incomplete component information

Sometimes it is impossible to construct the probability vector completely if some component probabilities are missing or only their bounds are known. In addition, if the conditional independence is not achievable for a system with statistically dependent components, one may afford to compute low-order joint probabilities only. Even in this case, the matrix-based system formulation still enables us to obtain the narrowest possible bounds on the probability of a system event by solving the LP problem

$$\begin{aligned} &\text{minimize (maximize) } \mathbf{c}^{\mathrm{T}}\mathbf{p} \\ &\text{subject to} \quad \mathbf{A}_1\mathbf{p} = \mathbf{b}_1 \\ &\qquad\qquad\quad \mathbf{A}_2\mathbf{p} \ge \mathbf{b}_2 \\ &\qquad\qquad\quad \mathbf{A}_3\mathbf{p} \le \mathbf{b}_3 \end{aligned} \quad (5)$$

where $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ denote the matrices whose rows are the event vectors for which exact probabilities or bounds are available; and $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ are the vectors of corresponding probabilities or bounds. This "LP bounds" method (Song and Der Kiureghian 2003a) has been successfully applied to structural systems, electrical substation networks and systems under stochastic excitations (Song and Der Kiureghian 2003a, 2003b, 2006).

2.6. *Conditional probability and importance measures*

In order to measure the relative importance of components or cut sets, many importance measures (IM) have been used in the system engineering community. Song and Der Kiureghian (2005) reviewed several IMs including Fussell-Vesely IM (Fussell 1973) and proposed a method to estimate narrow bounds on various IMs by the LP bounds method. It is proposed to use the conditional probability of the component event given the system failure as a useful component IM. The conditional probability IM (CIM) is computed as

$$CIM_i = P(E_i \mid E_{sys}) = \frac{P(E_i E_{sys})}{P(E_{sys})} \tag{6}$$

Most of IMs, including CIM, are defined as the ratio of the probability of a new system event E'_{sys} to that of the original system event E_{sys}. Therefore in the MSR formulation, an IM is computed by $(\mathbf{c}'^{\mathrm{T}}\mathbf{p})/(\mathbf{c}^{\mathrm{T}}\mathbf{p})$ where $\mathbf{c}'$ is the event vector of E'_{sys}. Note that once a system reliability analysis is performed by the MSR method, the only additional tasks are to find the corresponding $\mathbf{c}'$ by Eq. 2 and calculate $(\mathbf{c}'^{\mathrm{T}}\mathbf{p})/(\mathbf{c}^{\mathrm{T}}\mathbf{p})$.

3. Application I: connectivity of bridge network

The MSR method is applied to a traffic network that connects eight cities by highways with twelve bridges. Figure 1 shows the cities and the bridges in the network by circles and squares, respectively. It is assumed that the bridges are the only components of the highway system whose seismic damages may cause paths to be disconnected. City 1 has a major hospital that should be accessible from the other cities in case of emergency. For decision-makings on the retrofits of bridges or general hazard mitigation strategies, it is essential to estimate the probability of disconnection between cities and the critical facility based on the fragility estimates of bridge structures and a seismic hazard model. However, the events of disconnections are so complex that it is difficult to identify all the cut sets or link sets, and to compute the probability of disconnections analytically. This example demonstrates the merits of the proposed MSR method in identifying or handling various complex system events and estimating the probabilities thereof.

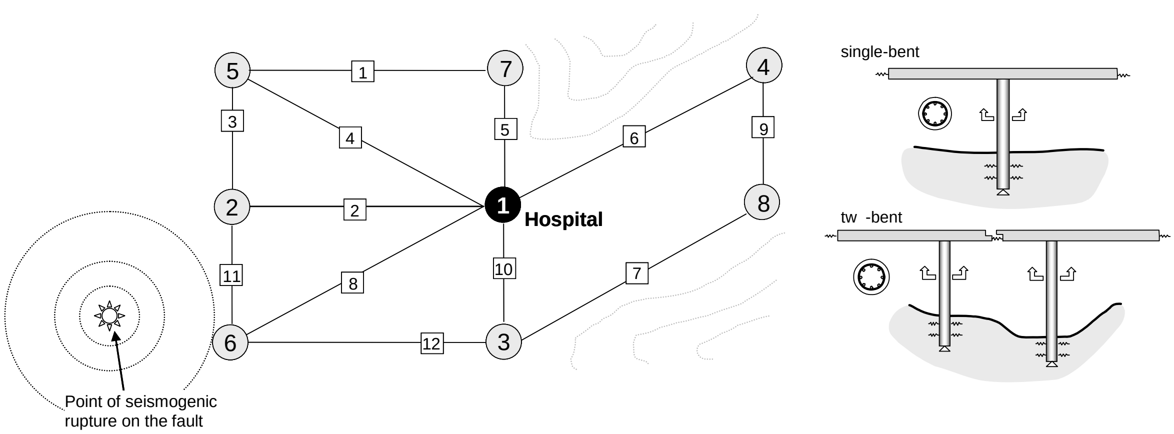

Figure 1: Example transportation network and bridge models considered (Kang et al. 2007)

For the bridge failure probabilities given a seismic intensity, we make use of predictive fragility estimates based on multi-variate probabilistic capacity and demand models developed by a Bayesian framework (Gardoni et al. 2002, 2003). We consider two bridge configurations, single-bent and two-bent overpasses shown in Figure 1. Details on the design parameters for the overpass bridges and the network are given in Kang et al. (2007) and Gardoni et al. (2003). We consider earthquakes that may occur at a point of seismogenic rupture on the nearby fault shown in Figure 1. The spectral accelerations at the bridge sites are estimated by an attenuation law by Campbell (1997).

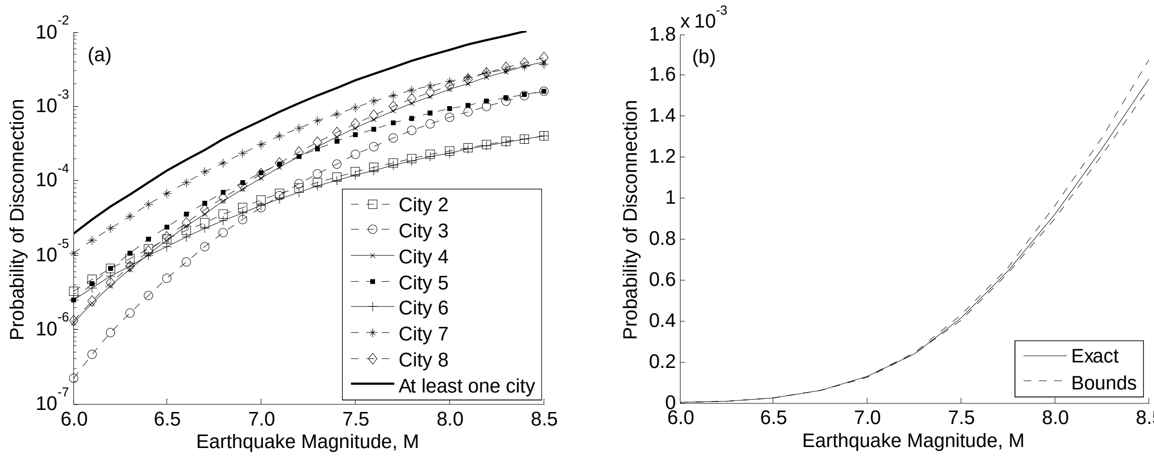

Figure 2: (a) Probability of disconnection between cities and hospital; and (b) bounds on probability of disconnection between City 5 and hospital

We first compute the probability of disconnection from the hospital for each city. The disconnection event is represented by a link set system event of bridge failures. The matrix-based framework allows us to identify the system event vector directly from matrix manipulations instead of identifying and handling numerous link sets. In this example, we assume the failures of bridges are conditionally independent of each other given an earthquake magnitude $M = m$. While there may exist dependencies due to commonalities in the experienced deteriorating conditions, maintenance and the uncertain seismic attenuations, etc., conditional independence of bridge failures given a seismic intensity is considered as a reasonable approximation. For a given magnitude $M = m$, we estimate the spectral acceleration at each bridge site and find the corresponding failure probability from the fragility function (Gardoni et al. 2003). Taking advantage of the conditional independence, we construct the conditional probability vector for a given earthquake magnitude $M = m$, denoted by $\mathbf{p}(m)$, using the matrix-based procedure in Eq. 3 and the bridge fragilities. Then, the conditional probability of the disconnection event given an earthquake magnitude is computed by Eq. 1. Note that the event vector is not affected by the magnitude and hence obtained only once. Figure 2a shows the conditional probabilities of disconnections for the eight cities considered. A decision-maker may be also interested in the probability that at least one city is disconnected from the hospital. This event is the union of the disconnection events of the cities. Therefore, the event vector for this new system event is easily obtained by Eq. 2 with the event vectors already identified for the cities. This probability is also shown in Figure 2a. The probability of disconnection for unknown earthquake magnitude can be obtained by Eq. 4 with the PDF of the magnitude.

Since the system event vector is independent of the probability calculations, one can easily estimate the probabilities of various system events simply by replacing the event vector. For example, Kang et al. (2007) estimated the probabilities of disconnections of counties consisting of a few cities, and the probabilities that a certain number of cities are disconnected from the hospital. The MSR framework can provide narrow bounds on the probabilities even if some of the fragilities are missing. Figure 2b shows the bounds on the probability of disconnection between City 5 and hospital in case the fragility of

Bridge 12 is not available, which are estimated by the LP bounds method in Eq. 5. Note that we can still estimate narrow bounds on the system probabilities despite the lack of complete information without introducing arbitrary assumptions.

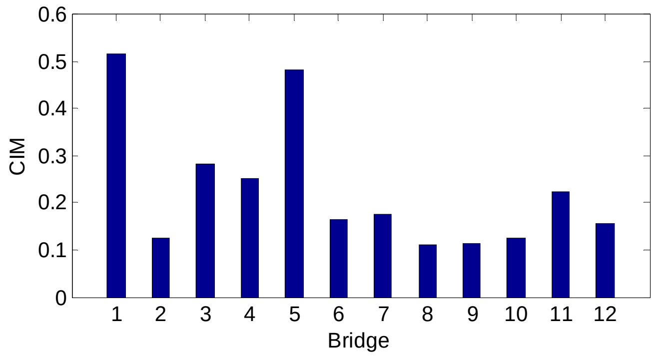

Figure 3: CIM of bridges with respect to likelihood of at least one disconnection

Suppose we intend to improve the post-hazard connectivity of the whole region by retrofitting selected bridges in the network. With limited budget, we may want to identify the bridges whose upgrade would enhance the connectivity in the most efficient manner. Here we define the important bridges in terms of the likelihood that there is at least one disconnected city in the region. Using the MSR method, we compute the importance measure CIM in Eq. 6 of each bridge for a given earthquake magnitude. By total probability theorem, we can evaluate CIM for an unknown magnitude. Figure 3 shows the CIM 's of the twelve bridges. Bridges 1 and 5 are identified as the most important ones. Note that these are on the paths connecting City 7, the most vulnerable city according to the results in Figure 2a. The MSR method allows us to compute the various importance measures based on different definitions of importance without additional cost in probability computation.

4. Application II: seismic damage of bridge structural system

Nielson (2005) developed analytical fragility curves for physical components of highway bridges such as bearings, columns and abutments. Assuming the capacity C_i and demand D_i of each physical component follow the lognormal distribution, the fragility of the *i*-th component is computed as follows by use of the safety factor $F_i = \ln C_i - \ln D_i$:

$$P(LS_i \mid IM) = P(F_i \le 0 \mid IM) = P(Z_i \le -\mu_{F_i} / \sigma_{F_i} \mid IM) = \Phi[-\mu_{F_i}(IM)/\sigma_{F_i}(IM)] \quad (7)$$

where LS_i denotes the event that the *i*-th component exceeds the given limit-state; μ_{F_i} and σ_{F_i} respectively denote the mean and standard deviation of the safety factor F_i ; $Z_i = (F_i - \mu_{F_i})/\sigma_{F_i}$; and $\Phi[\cdot]$ is the standard normal cumulative distribution function. Figure 4a shows the component fragility curves of a multi-span, simply-supported steel girder bridge based on Rix and Fernandez-Leon ground motion model. The event that at least one component fails is the union of the component failure events. In order to account for the statistical dependence between the demands of different components, D_i 's, Nielson (2005) performed Monte Carlo simulations.

We hereby compute the system fragility analytically by the MSR method. The correlation coefficient between Z_i and Z_j is

$$\rho_{Z_i Z_j} = \rho_{F_i,F_j} = \frac{(\zeta_{D_i} \cdot \zeta_{D_j})}{(\zeta_{C_i}^2 + \zeta_{D_i}^2)^{1/2} / (\zeta_{C_j}^2 + \zeta_{D_j}^2)^{1/2}} \cdot \rho_{\ln D_i, \ln D_j} \tag{8}$$

where $\rho_{\ln D_i, \ln D_j}$ is the correlation coefficient between the natural logarithms of the demands, which were reported in Nielson (2005); ζ_{D_i} and ζ_{C_i} are the standard deviations of $\ln D_i$ and $\ln C_i$, respectively. We find a DS-class correlation matrix (i.e, $\rho_{Z_i,Z_j} = r_i \cdot r_j$) that fits the correlation coefficients in Eq. 8 with the least square error and describe Z_i approximately as

$$Z_i = \sqrt{1 - r_i^2} U_i + r_i X, \quad i = 1, \ldots, n \tag{9}$$

where U_i, $i = 1, \ldots, n$ and X are statistically independent, standard normal random variables. In this example, the average of the errors is about 3% of the original coefficients. Since the demand is the only source of the statistical dependence, the component failures are conditionally independent of each other given the outcome of X. Therefore, one can compute the system fragility by the MSR method using Eq. 4 with $\mathbf{X} = \{X\}$. This system fragility curve is shown in Figure 4a. By simply replacing the event vector, the probability that at least k components fail, $k = 1, \ldots, 8$. is obtained and shown in Figure 4b. The importance measures of the bridge components at different seismic intensity levels can be computed easily as well.

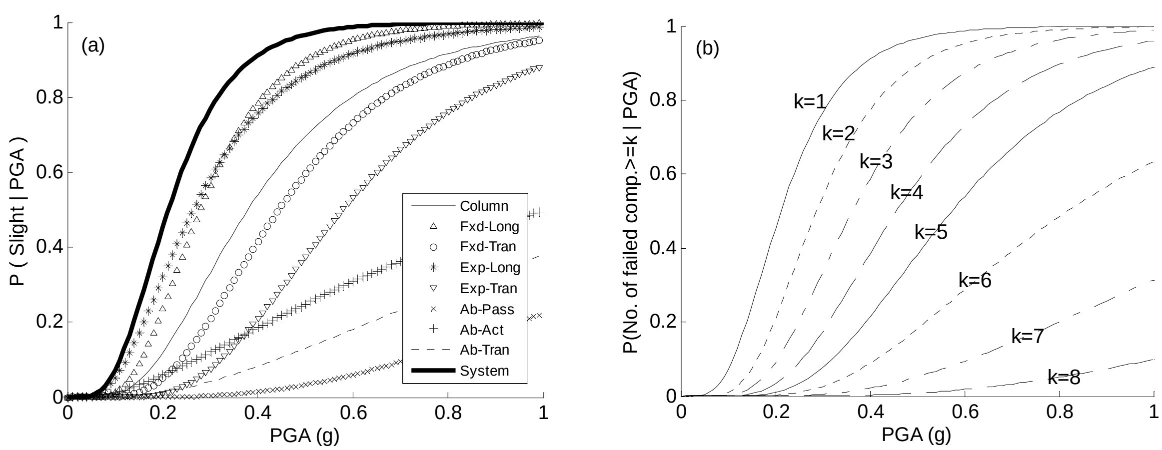

Figure 4: (a) Fragility curves for bridge components and system; (b) probability that at least k bridge components fail, $k = 1, \ldots, 8$.

5. Application III: progressive failure of truss structure

Consider a statically indeterminate truss structure in Figure 5, whose members are all perfectly brittle (Song and Kang 2007). The member force capacities C_i, $i = 1, \ldots, 6$ are assumed to be equally-correlated normal random variables, whose parameters are given in Figure 5a. Since we have a DS-class correlation matrix, the statistical dependence between member capacities are represented by a single random variable X as in Eq. 9. This structure collapses when at least two members fail. The complexity of this system problem arises from the fact that the load is re-distributed after member failures and lead to new component failure events. For example, the numbers in Figure 5b are the indices of the component failure events in the original structure and the structures with one failed member. This is a complex system problem in which the *reliability* of the structure is computed as

$$P(\overline{E}_{sys}) = P[\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6 \cup (E_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6)(\overline{E}_7\overline{E}_8\overline{E}_9\overline{E}_{10}\overline{E}_{11}) \\ \cup (\overline{E}_1 E_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6)(\overline{E}_{12}\overline{E}_{13}\overline{E}_{14}\overline{E}_{15}\overline{E}_{16})\cdots\cup(\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5 E_6)(\overline{E}_{32}\overline{E}_{33}\overline{E}_{34}\overline{E}_{35}\overline{E}_{36})] \quad (10)$$

This is a general system with 36 components. Due to the mutual exclusiveness of the sub-parallel systems, it is converted to seven smaller system problems:

$$P(\overline{E}_{sys}) = P(\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6) + P(E_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6\overline{E}_7\overline{E}_8\overline{E}_9\overline{E}_{10}\overline{E}_{11}) \\ \cdots + P(\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5 E_6\overline{E}_{32}\overline{E}_{33}\overline{E}_{34}\overline{E}_{35}\overline{E}_{36}) \quad (11)$$

We can further reduce the number of components in the last six systems in Eq. 11 to six by considering the perfect correlation between two component events given for the same member, e.g., $\overline{E}_2$ and $\overline{E}_7$. Figure 6 plots the conditional probabilities of the collapse for given external loads, computed by the MSR method. This result is successfully verified through comparison with 300 Monte Carlo simulations at each of the selected load levels.

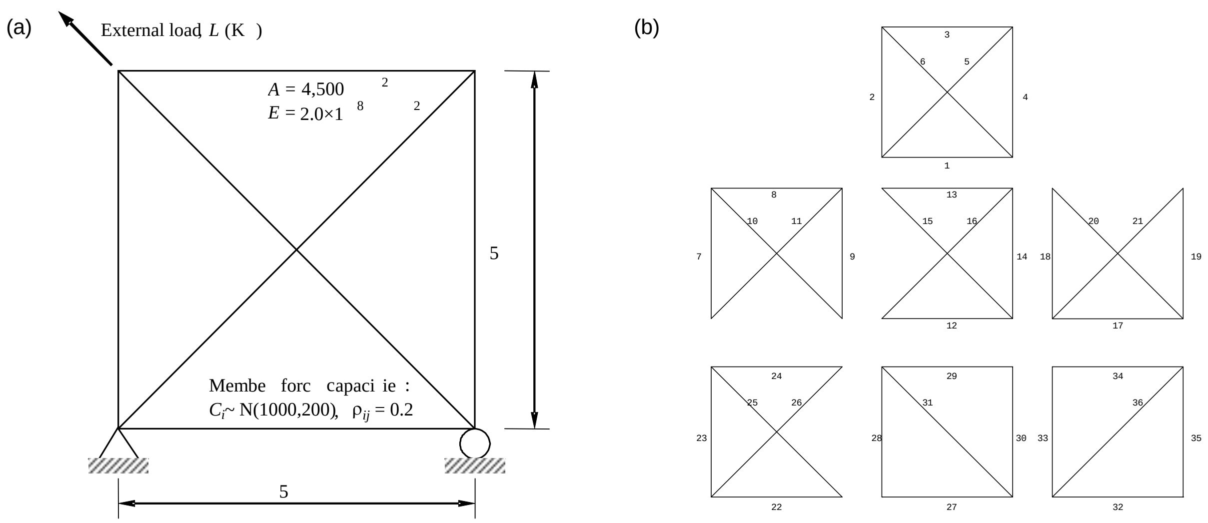

Figure 5: (a) Example truss structure; and (b) component failure events

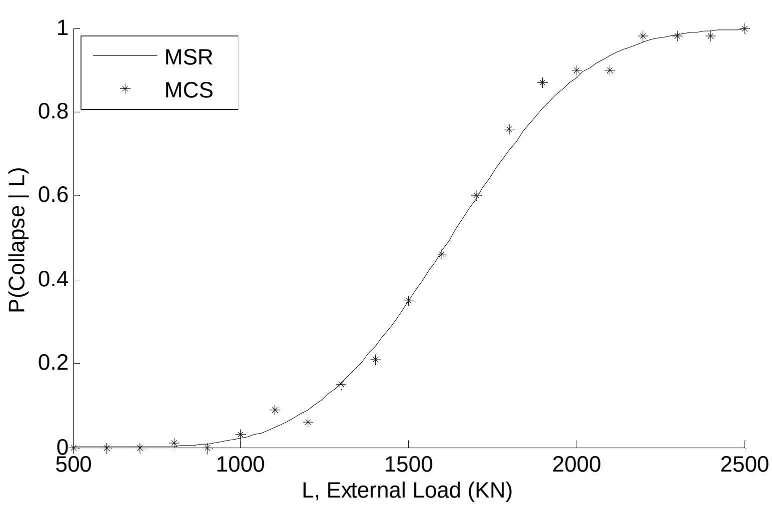

Figure 6: Conditional probability of collapse given external load

Conclusions

Using the Matrix-based System Reliability (MSR) method, the probabilities of complex system events can be estimated by simple matrix calculations. Matrix-based computing languages and software enable us to efficiently identify the matrices that represent the system event and the component risks. In case there exists statistical dependence between component failure events, we make use of conditional independence of components given outcomes of random variables representing common source effects. The matrix-based framework also helps obtain the narrowest bounds on the system failure probability when component information is not complete. The MSR method can also estimate various importance measures and conditional probabilities that help quantify the relative contribution of components to the system events of interest. These merits of the MSR method are successfully demonstrated by its applications to a transportation network, a bridge structure system and the progressive failure of a truss structure.

Acknowledgements

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