Risk Measures Beyond Expected Cost…

Unified Reliability and Design Optimization in Earthquake Engineering

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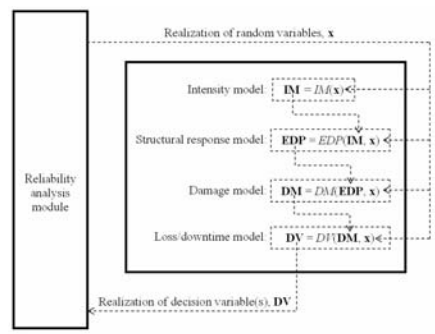
Outline

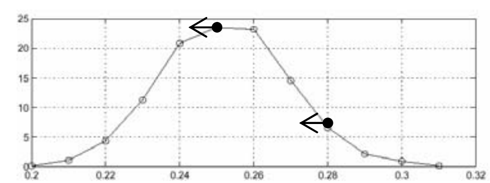
- **Trends**

$$\lambda(dv) = \int_0^\infty \int_0^\infty \int_0^\infty G(dv \mid dm)\, |dG(dm \mid edp)|\, |dG(edp \mid im)|\, |d\lambda(im)|$$

- **"Unified" reliability analysis**

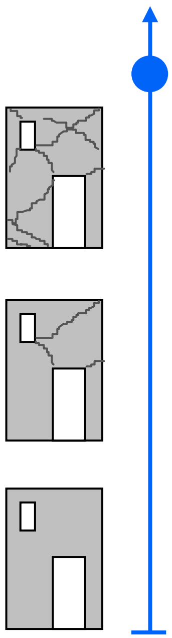


- **Design optimization**



Trends

Structural response



Collapse ←
- Addressed by design codes
- Causes of failure: human error, omissions

Damage
- Damage is *implied* in seismic design
- Considerable concern to stakeholders
- Renaissance for reliability analysis

Pacific Earthquake Engineering Research Center (PEER)

ATC-58

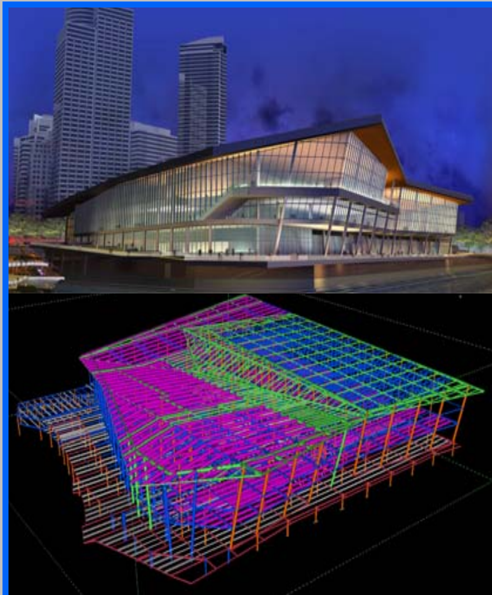
Evaluation of the "PEER Equation"

$$\lambda(dv) = \int_0^\infty \int_0^\infty \int_0^\infty G(dv \mid dm)\, \left|dG(dm \mid edp)\right|\, \left|dG(edp \mid im)\right|\, \left|d\lambda(im)\right|$$

- *Moehle, Stojadinovic, Der Kiureghian, Yang (2005)*
- *Tony Yang, Ph.D. thesis, UC Berkeley (2006)*

- Hazard curve for intensity measure S_a
- Suite of ground motions, scaled to match S_a
- Nonlinear dynamic analysis
- Joint lognormal structural response vector
- Probability curves for damage states DS: $P(DS \geq DS_i | x_j)$
- Associated repair costs from lookup tables

@ UBC



Treatment of epistemic uncertainty
Koduru, Haukaas

Cost-benefit importance measures
Riederer, Haukaas

Finite element reliability analysis
Koduru, Haukaas

Reliability-based design optimization
Liang, Haukaas, Royset

Response sensitivities
Haukaas, Scott

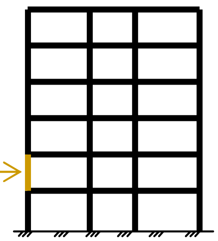
Second-order response sensitivities
Bebamzadeh, Haukaas

Model errors
Bebamzadeh, Haukaas, Gardoni

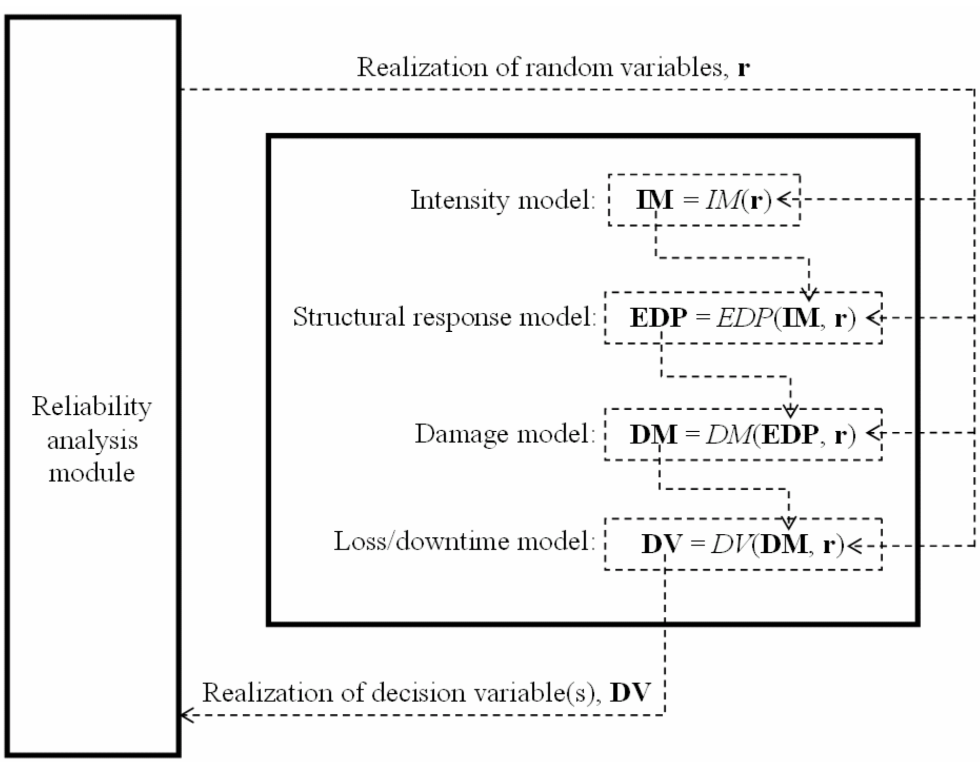
Probabilistic models
Zhu, Elwood, Haukaas, Gardoni

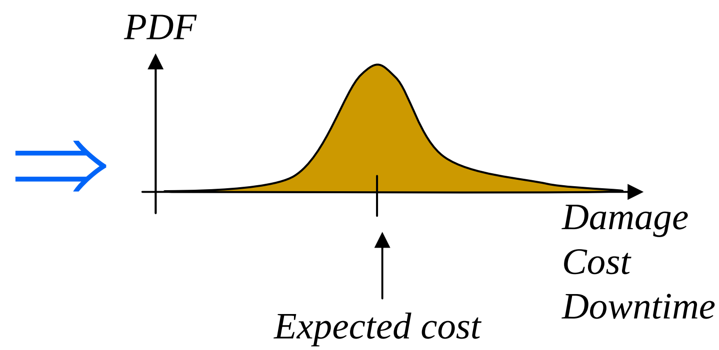
Unified Reliability Analysis

- **Reliability formulation** --------------- $p = \int \cdots_{g(\mathbf{r}) \le 0} \int f(\mathbf{r}) d\mathbf{r}$
- **Explicit probabilistic models** ---------- $\ln(\delta_s) = \ln\left(\theta_4 \rho'' + \theta_6 \frac{s}{d} + (0.0069 - 0.26\theta_6)\frac{a}{d} + \theta_9 \frac{P}{A_g f'_c}\right) + \sigma\varepsilon$ →



- **Limit-state function** ------------------ $g(\mathbf{r}) = dv - DV = dv - DV(\mathbf{r}, DM(\mathbf{r}, EDP(\mathbf{r}, IM(\mathbf{r}))))$





Full-scale Examples

- Koduru & Haukaas, ICASP10

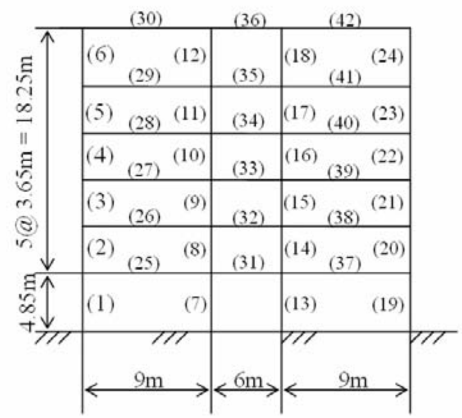


Figure 7. Finite element model of a six storey structure located in Vancouver, Canada. Element numbers are provided in parenthesis.

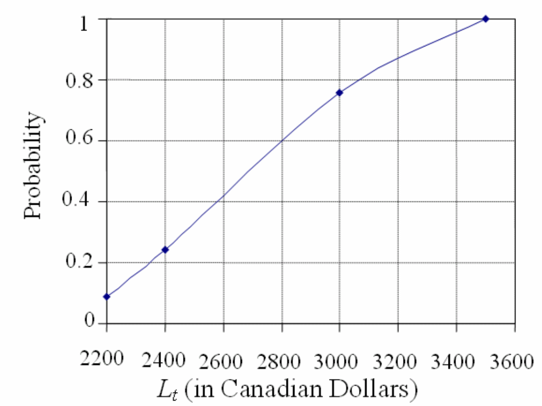


Figure 8. Probability distribution of total structural loss.

- Koduru & Haukaas, COMPDYN'07

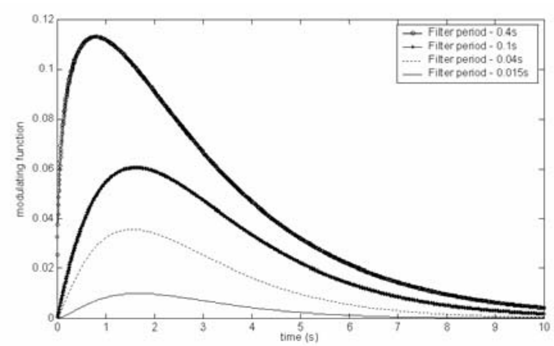


Figure 4: Modulating functions.

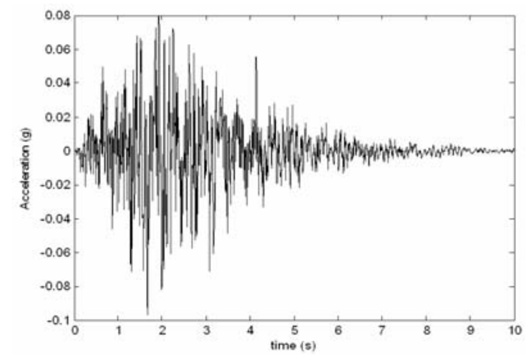
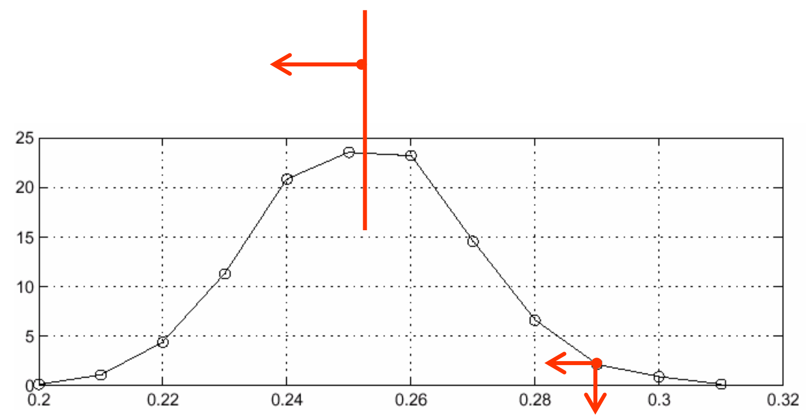


Figure 5: Sample realization of the ground motion.

Design Optimization

- **Expected cost approach:** Criticized because it does not explicitly address the unlikely but potentially devastating costs in the tail of the cost distribution (Haimes 1998)

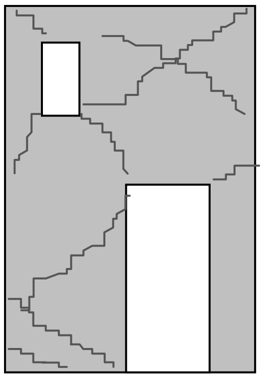
- **Proposal:** Perform the decision making in the tail of the distribution



- **Utilizing unified reliability analysis:** $g(\mathbf{d},\mathbf{r}) = c_o - c_c(\mathbf{d},\mathbf{r}) - c_d(\mathbf{d},\mathbf{r})$

  - $\mathbf{d}$: *design variables*
  - $\mathbf{r}$: *random variables*
  - c_o: *cost threshold*
  - c_c: *cost of construction*
  - c_d: *losses in earthquake*

Exceedingly Simple Example to Illustrate the Optimization Methodology



⇒

Equivalent stiffness, k ← Design variable
Applied load, P ← Random variable
Displacement response $u=P/k$ ← Response that causes damage

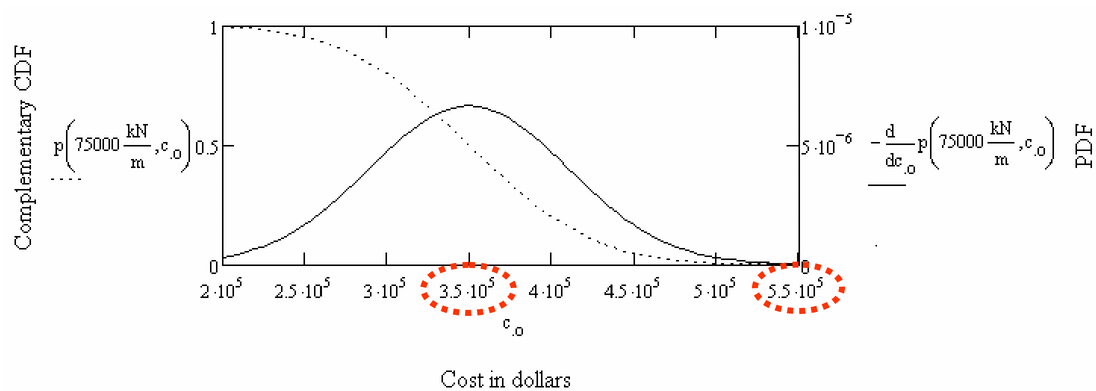
Cost of damage: $c_d(P,k)=m_u u$
Cost of increasing the stiffness: $c_c(k)=m_k k$

$\mu_P=15{,}000kN$ m_u = \$10,000 per cm
σ_P=4,500kN m_k = \$2 per kN/m

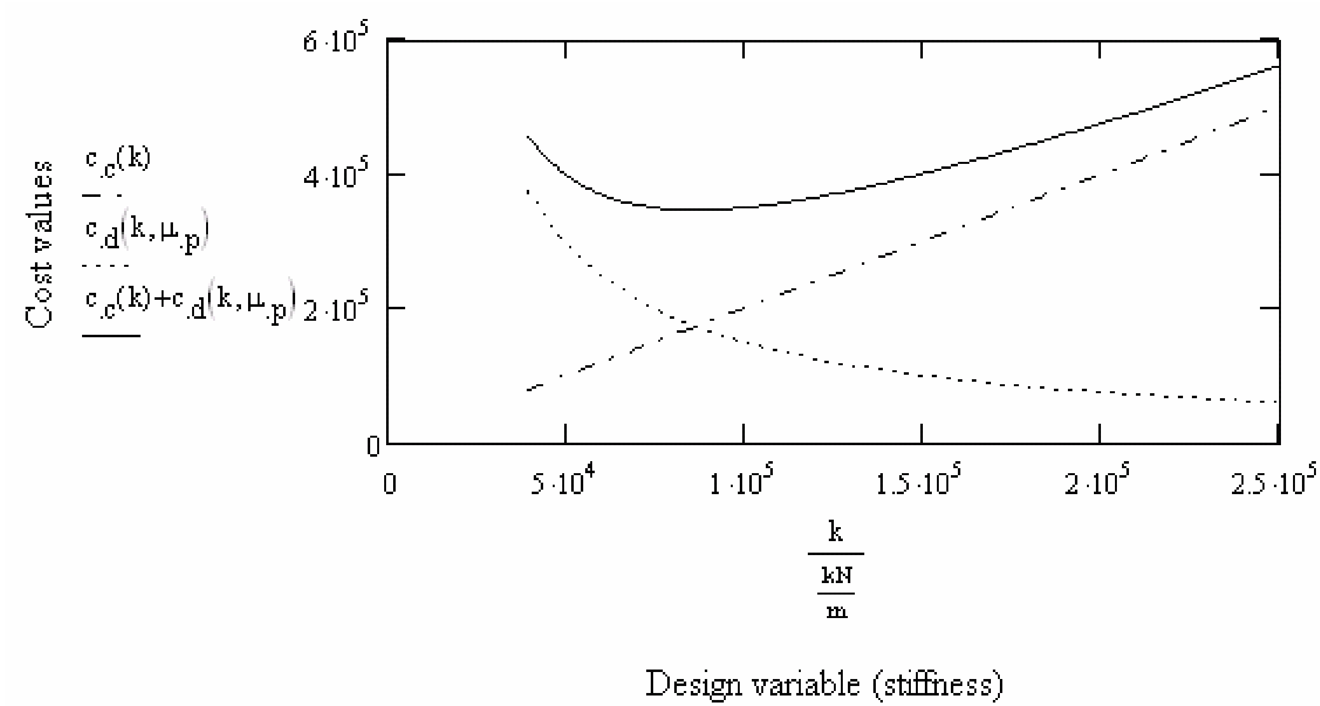
- **Unified reliability analysis**

$$\beta(k,c_o)=\text{straightforward}$$

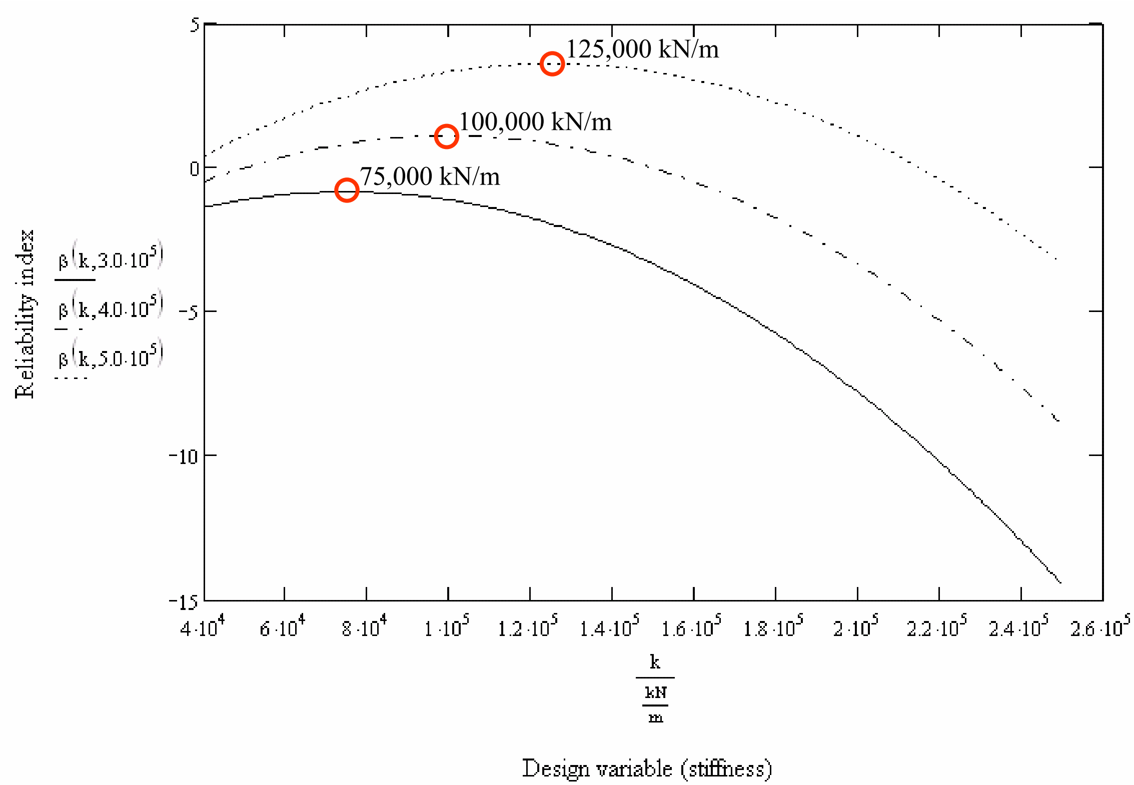
$$p(k,c_o)=\Phi(-\beta(k,c_o))$$



Construction cost and damage cost at the mean value of the random load, as function of the design variable:



Reliability index associated with three different cost thresholds, plotted as function of the design variable:

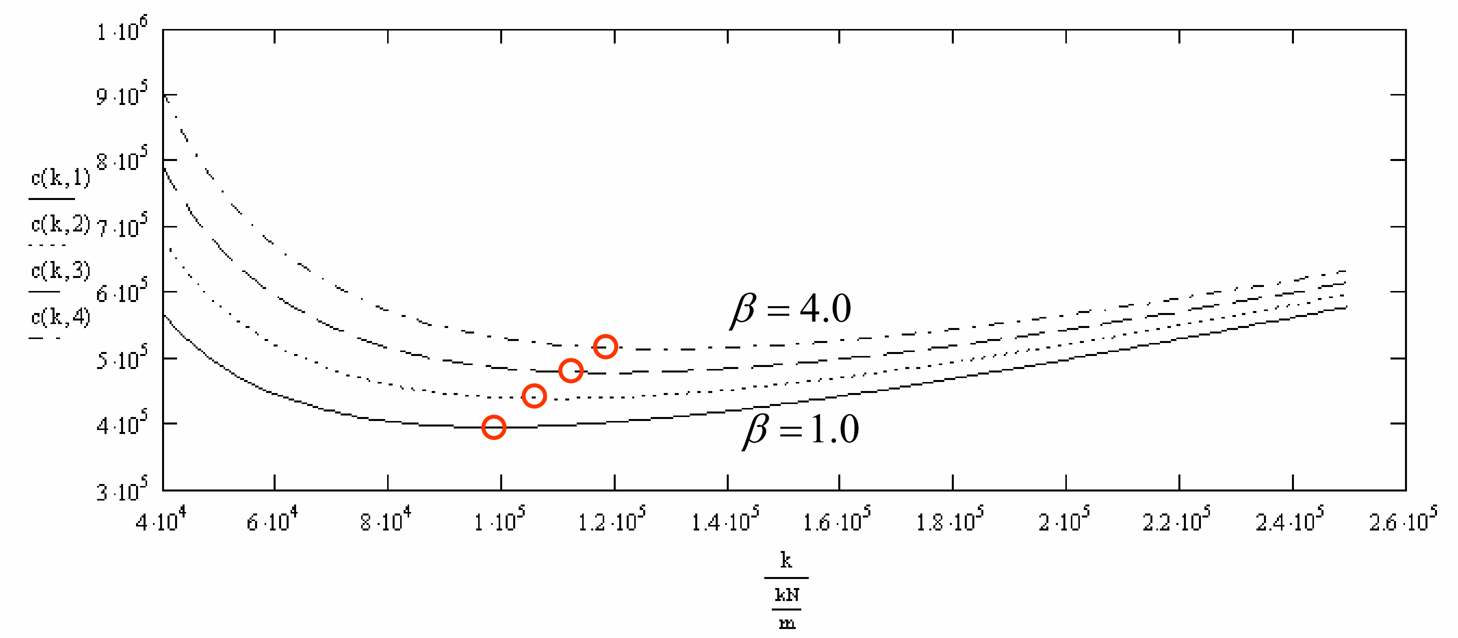


- **Observations:** The optimal value of the design variable increases with increasing cost threshold

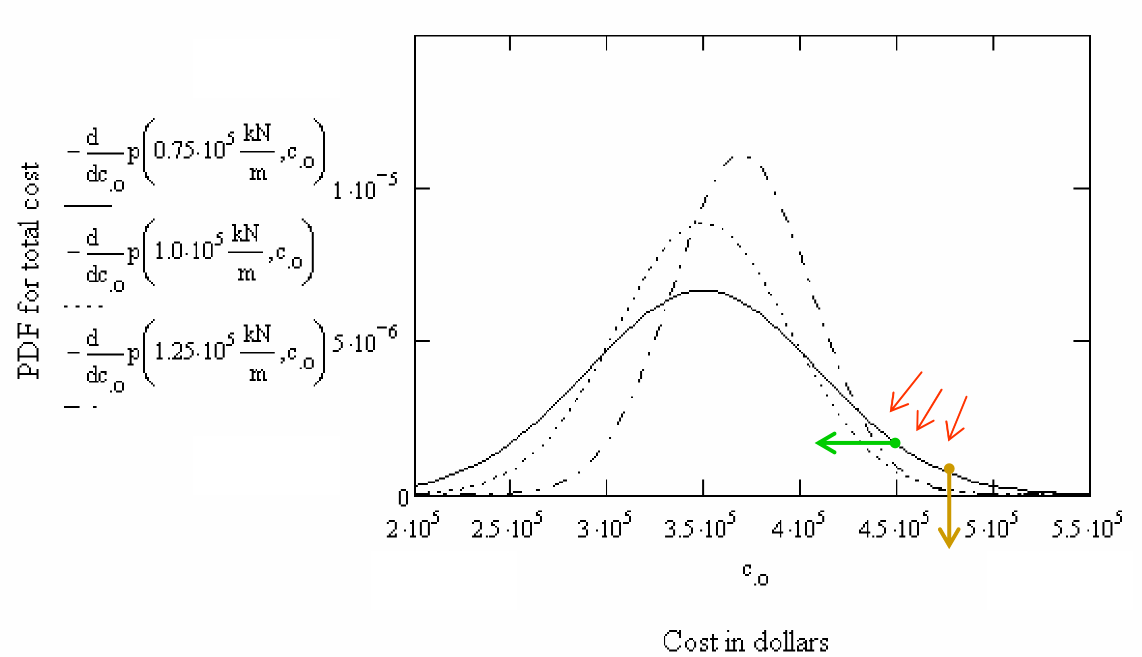
  → Risk averseness on the part of the decision maker becomes part of the design

  Akin to a utility function with built-in risk averseness

Total cost threshold associated with four different reliability indices, plotted as function of the design variable:



PDF for total cost, associated with different optima:



- **Possible problem formulations:**

  Minimize exceedance probability at a cost threshold: $\min\{p(\mathbf{d})\,|\,c_o\}$

  Minimize cost threshold at a probability: $\min\{c_o(\mathbf{d})\,|\,p\}$

Thank You for Your Attention