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Cost and benefit including value of life and limb measured in time units

Ove Ditlevsen and Peter Friis-Hansen
Technical University of Denmark

Standard expected loss function example

$$L(\lambda_1, \ldots, \lambda_n, \mu_1, \ldots, \mu_n) = c(\lambda_1, \ldots, \lambda_n, \mu_1, \ldots, \mu_n) + \frac{1}{\gamma} \sum_{i=1}^{n} \lambda_i \mu_i - \frac{g}{\gamma}$$

Preference equilibrium between work time and free time in good health

Time value modeling

$$G = pS = pw(S/w)$$

Weber-Fechner law

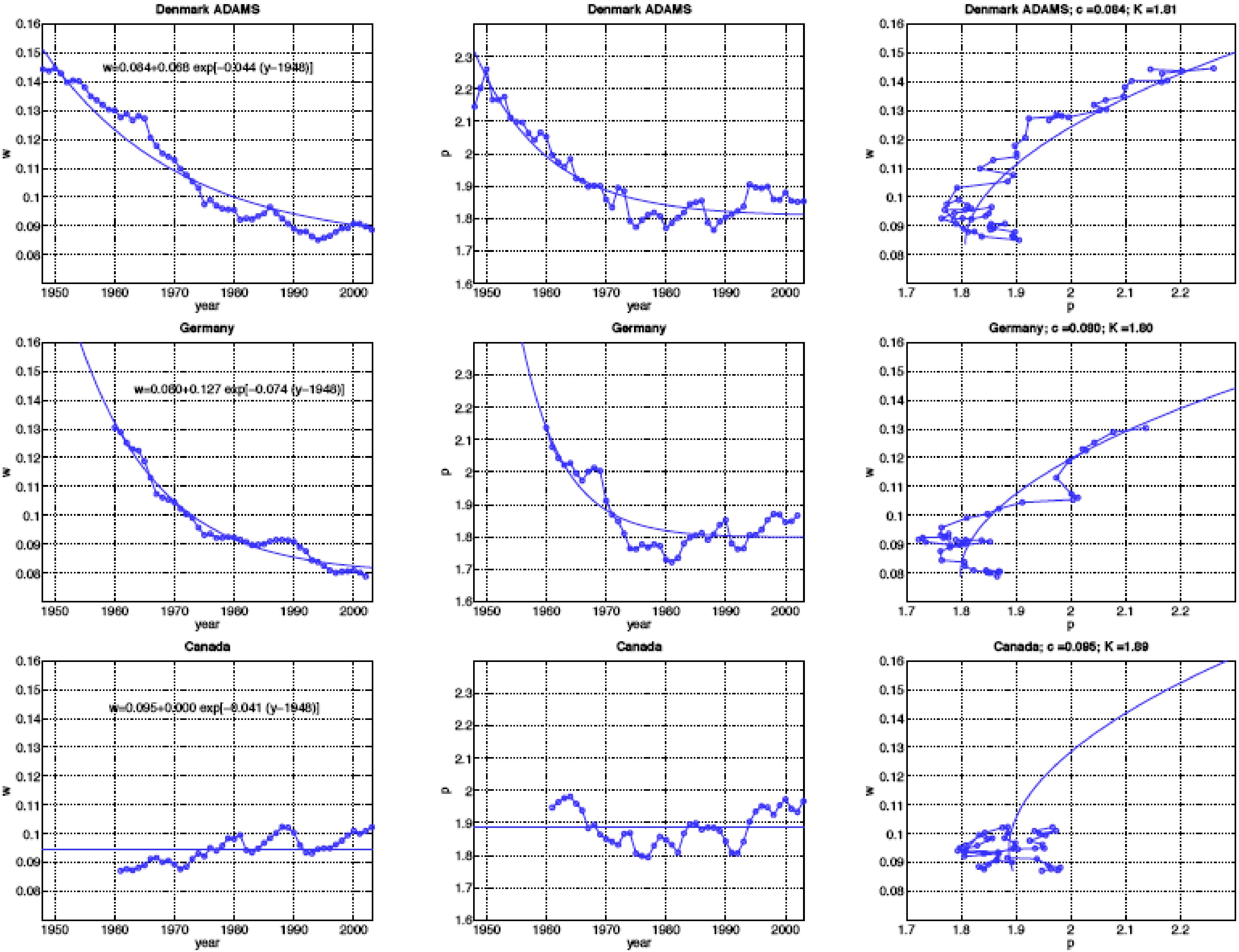
logarithmic relation

$$c(r)\frac{\mathrm{d}(pw/r)}{pw/r} + [1 - c(r)]\frac{\mathrm{d}(1 - w/r)}{1 - w/r} = 0$$

$$\left(\frac{c(r)}{w/r} - \frac{1-c(r)}{1-w/r}\right)\mathrm{d}(w/r) = -\frac{c}{p}\,\mathrm{d}p$$

$$p(w/r,r) = p_{\min}\frac{(1-w/r)^{1-1/c(r)}\,(w/r)^{-1}}{(1-c(r))^{1-1/c(r)}\,c(r)^{-1}}$$

$$w = c + a\, e^{-b(t-1948)} \to c \text{ as } t \to \infty$$



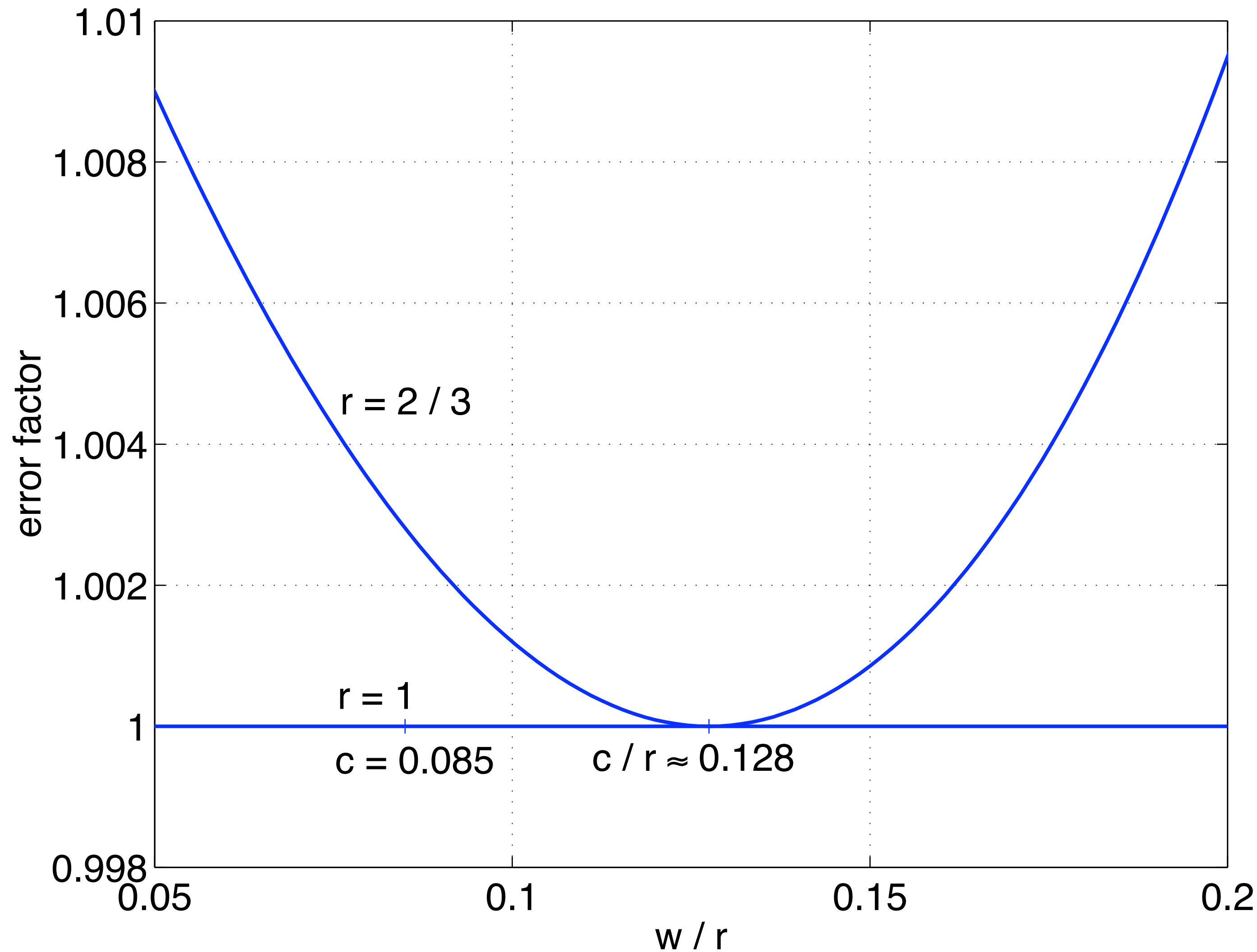
The time reduction factor r and the wealth index I_r

$$\frac{\mathrm{d}I_r}{I_r} = V(r)\left[c(r)\frac{\mathrm{d}(pw/r)}{pw/r} + [1 - c(r)]\frac{\mathrm{d}(1 - w/r)}{1 - w/r}\right] \quad (6)$$

$$I_r = J(r)\left[\left(p\frac{w}{r}\right)^{c(r)}\left(1 - \frac{w}{r}\right)^{1-c(r)}\right]^{V(r)}$$

$$= J(r)\exp\left\{V(r)\left[c(r)\log\left(p\frac{w}{r}\right) + [1 - c(r)]\log\left(1 - \frac{w}{r}\right)\right]\right\} \quad (7)$$

$$c(r_0) = c / r_0$$



1.01
1.008
1.006
1.004
1.002
1
0.998
0.05
0.1
0.15
0.2
error factor
w / r
r = 2 / 3
r = 1
c = 0.085
c / r ≈ 0.128

Invariance principle

$$\text{experienced worth of } \mathrm{d}(rE) = V(r)\frac{\mathrm{d}(rE)}{rE} = V(r)\left[\frac{\mathrm{d}r}{r} + \frac{\mathrm{d}E}{E}\right] \tag{8}$$

differentiation of (7) under constant w

$$\frac{\mathrm{d}I_r}{I_r} = V(r)c(r)\frac{\mathrm{d}p}{p} + V(r)\left\{c'(r)\log\left(\frac{pw}{r-w}\right) - c(r)\frac{1}{r-w} + \frac{w}{r(r-w)}\right.$$
$$\left. + \frac{V'(r)}{V(r)}\left[c(r)\log\left(p\frac{w}{r}\right) + [1-c(r)]\log\left(1-\frac{w}{r}\right)\right] + \frac{J'(r)}{J(r)}\right\}\mathrm{d}r \tag{9}$$

adding (8) and (9) and setting to zero

$$-\frac{\mathrm{d}p}{p} = \frac{1}{c(r)}\frac{\mathrm{d}E}{E} + \left[\frac{1-c(r)}{r-w} + c'(r)\log\left(\frac{pw}{r-w}\right)\right]\frac{\mathrm{d}r}{c(r)} \tag{10}$$

$$\mathrm{d}(I_r rE)/(I_r rE) = 0$$

Life Quality Time Allocation Index (LQTAI) $I_r rE$

For the actual
value r_0 of r and the stationary state where $w = c$ and $p = p_{\min}$

$$-\frac{\mathrm{d}p}{p_{\min}} = \frac{r_0}{c}\frac{\mathrm{d}E}{E} + \left[\frac{1}{c} + \frac{r_0}{c}c'(r_0)\log\left(\frac{p_{\min}c}{r_0 - c}\right)\right]\mathrm{d}r$$

$$\frac{r_0}{c}c'(r_0) = -\frac{1}{r_0}$$

postulating that p is not influenced by variations of r

Public acceptance rules

$$\sum_{i=1}^{n}(g_i - \lambda_i \mu_{oi})$$

$$\sum_{i=1}^{n} g_i = g$$

$$g_1/\mu_{o1} = g_2/\mu_{o2} = \ldots = g_n/\mu_{on}$$

$$(g_i - \lambda_i \mu_{oi})\rho \geq \lambda_i \mu_{pi}$$

$$\frac{\lambda_i \mu_{oi}}{g_i} \leq \left(1 + \frac{1}{\rho}\frac{\mu_{pi}}{\mu_{oi}}\right)^{-1}$$

A model for determining dE and dr generated by an accident

$$L = \begin{cases} \min\{X, Y\} \\ Y \end{cases} \quad \text{and} \quad L_{\mathrm{h}} = \begin{cases} R\min\{X, Y\} & \text{both with probability } P_{\mathrm{f}} \\ RY - T\mathbf{1}_{X<Y} & \text{both with probability } 1 - P_{\mathrm{f}} \end{cases}$$

$$E[\mathbf{1}_{X<Y}] = P(X < Y) = \kappa E$$

$$\mathrm{E}[L] = P_{\mathrm{f}}\frac{1}{\kappa}\int_0^\infty (1 - e^{-\kappa t}) f_Y(t)\,\mathrm{d}t + (1 - P_{\mathrm{f}})\mathrm{E}[Y]$$

$$\mathrm{E}[L_{\mathrm{h}}] = r\mathrm{E}[L] - (1 - P_{\mathrm{f}})\mathrm{E}[T]E[\mathbf{1}_{X<Y}] = (r + \mathrm{d}r)E$$

$$\frac{\mathrm{E}[Y] - \mathrm{E}[L]}{\kappa\mathrm{E}[L]^2} \to P_{\mathrm{f}}\frac{1}{2}(1 + V_Y^2) \quad \text{or} \quad \frac{\mathrm{d}E}{E} \approx -P_{\mathrm{f}}\frac{1}{2}(1 + V_Y^2)\,\kappa E$$

$$\mathrm{d}r = -(1 - P_{\mathrm{f}})\kappa\mathrm{E}[T]$$

Distribution on accident categories

N_i persons are assigned to category $i = 1, \ldots, n$

$$N_1 + \ldots + N_n = N$$

$$N\,\mathrm{d}p = N_1(\mathrm{d}p)_1 + \ldots + N_n(\mathrm{d}p)_n$$

time allocation per accident suffering person in category i

$$\frac{(\mathrm{d}p)_i}{\kappa_i} = \frac{r_0}{c}\frac{1}{2}(1 + V_Y^2)\,E p_{\min} P_{\mathrm{fi}} + \left[\frac{1}{c} + \frac{1}{r_0}\log\left(\frac{r_0 - c}{p_{\min} c}\right)\right]\mathrm{E}[T_i]\,p_{\min}(1 - P_{\mathrm{fi}})$$

Example: Ferry on fire

$$\frac{\lambda_i \mu_{\mathrm{o}i}}{g_i} \leq \left(1 + \frac{1}{\rho}\frac{\mu_{\mathrm{p}i}}{\mu_{\mathrm{o}i}}\right)^{-1}$$

$$\frac{1.2 \cdot 10^{-2} \cdot 0.0246}{0.3 \cdot 3.075} = 3.20 \cdot 10^{-3} \leq \left(1 + \frac{1}{\rho}\frac{22.2 - 0.0246}{0.0246}\right)^{-1} = 3.32 \cdot 10^{-3}$$

waste of your valuable time?

Results: The annual frequency of fire in Scandinavian waters is estimated to $\kappa = 1.2 \cdot 10^{-2}\,\mathrm{y}^{-1}$. Given the occurrence of a fire estimates of the probability of a fatal accident and of injury is $P_\mathrm{f} = 7.85 \cdot 10^{-3}$ and $P_\mathrm{i} = 0.214$, respectively. Thus, the probability is 0.778 that the fire ocurrence does not harm an arbitrary passenger or crew member. Conditional on injury the expected recovery time is estimated to $\mathrm{E}[T \mid \mathrm{injury}] = 0.27$ y. Hence, $E[T] = (0.778 \cdot 0 + 0.214 \cdot 0.27)/(0.778 + 0.214) = 0.059$ y $= 22$ days. Inserting these values into (19) and using $c = 0.084$, $p_\mathrm{min} = 1.81$, $r_0 = 0.95$, $E = 80$ y, and $V_Y = 0.2$, the two terms in (19) become $\mathrm{d}p_\mathrm{f}/\kappa = 6.68$ y and $\mathrm{d}p_\mathrm{i}/\kappa = 1.34$ y. Multiplying these by $S = GDP/p_\mathrm{min} = 33,340/1.81 = 18,420$ €/y their monetary equivalents are 0.123 mill € and 0.0248 mill €, respectively. Thus the societal cost of an injury is about 20% of the cost of a fatality. For $N = 150$ passengers the total expected socio-economic loss of life and limb per fire becomes $150 \cdot (0.123 + 0.025) = 22.2$ mill €.

The owner loss expectations are estimated to 100.000 € / fatality and 4.000 € / injury. The expected owner loss due to fatalities and injuries caused by a fire occurrence is thus $\mu_\mathrm{o} = (7.85 \cdot 10^{-3} \cdot 0.1 + 0.214 \cdot 0.004) \cdot 150 = 0.0246$ mill €. The total annual gain before tax for the RoRo-ferry is 3.075 mill € / y of which 30% is allocated to cover the fire risk. Inserting these values into the acceptance rule (13) using $\rho = 0.3$ gives

$$\frac{1.2 \cdot 10^{-2} \cdot 0.0246}{0.3 \cdot 3.075} = 3.20 \cdot 10^{-3} \leq \left(1 + \frac{1}{\rho}\frac{22.2 - 0.0246}{0.0246}\right)^{-1} = 3.32 \cdot 10^{-3} \tag{20}$$

Thus the public acceptance criterion (13) is satisfied. For $\rho = 1$ the criterion is the so-called LQI (or LQTAI) criterion which is less restrictive than if $\rho < 1$. For $\rho = 1$ the right side of (20) becomes $1.10 \cdot 10^{-2}$.

Nature preservation willingness index

$$W(\mathbf{x},\mathbf{y}) = I(\mathbf{x},\mathbf{y})W$$

$$W = \mathrm{GDP} \times Q$$

$$Q = T/E$$

$$\mathrm{d}p/p = -\mathrm{d}Q/Q$$

$$\mathrm{d}[I(\mathbf{x},\mathbf{y})p] = -I(\mathbf{x},\mathbf{y})p\,\mathrm{d}Q/Q$$

$$\mu_{\mathrm{p}} = \gamma^{-1}(1 - e^{-\gamma E|\mathrm{d}Q|})p_{\min}\frac{|\mathrm{d}Q|}{Q}\int_{\text{all }\mathbf{x}} d(\mathbf{x})\,I[\mathbf{x},\mathbf{y_0}(\mathbf{x})]\,\mathrm{d}\mathbf{x}$$

$$I(\mathbf{x},\mathbf{y}) = \exp[-|\mathbf{x}-\mathbf{y}|^2/(2\sigma^2)]$$