Risk Communication with Generalized Uncertainty and Linguistics

Ross B. Corotis, NAE
Denver Business Challenge Professor of Engineering
University of Colorado at Boulder

Introduction

- Determining acceptable risk is necessary for optimal allocation of resources
- Social psychology research provides extensive information on risk avoidance
- Bringing linguistically-based data into risk analysis is not straightforward
- Generalized information theory holds several advantages over basic probability

Generalized Uncertainty

- Probability measures are controlled by standard axioms such as $P[A \cup b] = P[A] + P[B]$
- But we can replace probability by any monotone measure, μ, such that $\mu[A \cup B] \leq\equiv\geq \mu[A] + \mu[B]$
- P[Ø] = 0 = μ[Ø] still holds, but P[S] = 1 becomes
  $$\forall \ A, B \in C, \ if \quad A \subseteq B, \ then \ \mu[A] \leq \mu[B]$$
- This generalization permits the use of imprecise numbers in place of precise probabilities (Klir 2006)

Fuzzy Sets

- Social psychologists' risk perception terms fit the framework of fuzzy set membership

$$A = \langle x | A(x) \geq \alpha \rangle$$

- This cut-set membership definition allows the use of classical probability algebra
- Uncertainties in linguistic definition must be distinguished from epistemic uncertainty

Fuzzy Sets

- Fuzzy set calculus replaces $E[Y] = \int g(x) f_X(x) dx$ by the membership relation $P[A] = \int A(x) f_X(x) dx$
- Some basic classical probability measures are unchanged, such as $P[A \cup B] = P[A] + P[B] - P[A \cap B]$
- Whereas other properties are not, such as

$P[A \cup \bar{A}] \leq 1$ and $P[A \cap \bar{A}] \geq 0$

Alternate Measures of Uncertainty

- Monotonic rather than linear measures allow graduations in membership belief
- Belief measures can sum or integrate to more or less than unity over all values
- Upper and lower probabilities result from the integral values of beliefs
- Belief measures can be applied to both crisp and fuzzy sets

Alternate Measures of Uncertainty

- Shannon's (1948) entropy allows generalized uncertainty based on principles of minimization or maximization

$$H = -a\int f_X(x)\log_b(x)dx$$

in which we usually use a=1 and $\log_b$ = ln

- Entropy can be used for maximum value from linguistic variables of multiple experts

Built Environment Risk Factors

- Social psychology work of Slovic (2004) and others supplements probability and consequence with dread, voluntariness, trust, equitability and affective* feeling

  * quality of "goodness" or "badness"

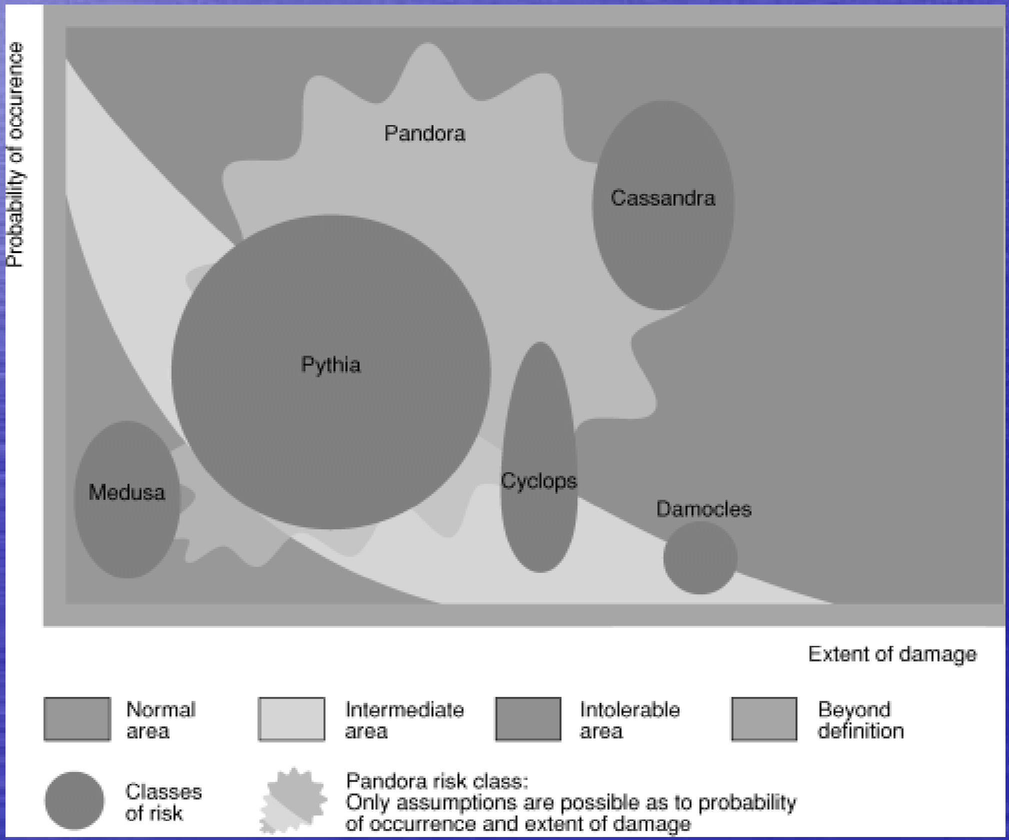
- Framing of issues (Kahnemann & Tversky 2000) must be compensated

Perceived Risk and Benefit for Common Hazards, on a scale of 0-100 (adapted from Slovic, 2000)

Hazard Category	Number	Perceived Risk		Perceived Benefit	
		Average	Std Dev	Average	Std Dev
All Hazards	90	39	17	46	17
Physical Environment	12	33	9	51	12
Transportation	8	35	10	51	15
Power Generation	5	36	22	60	15
Health	35	44	15	43	16
Society	7	70	8	24	19
Job Related	4	41	8	62	23
Science	4	33	10	45	9
Recreation	15	25	5	45	10

Linguistic Risk Assessment

- Klinke and Renn (2002) Greek Mythology



Generalized Uncertainty and Linguistics

Consider a city with the following hazards:

Geophysical (G)
Cimatological (C)
Intentional (I)

Assume the true (but unknown) risks conditioned on an some event are 50%, 50% and 0%, respectively.

Example Uncertainty Measures

Source	Probability
G	0.5
C	0.5
I	0.0
GUC	1.0
GUI	0.5
CUI	0.5
GUCUI	1.0

Two Expert Panels Interviewed

Source	Probability	Panel 1 Belief	Panel2 Belief
G	0.5	5%	15%
C	0.5	5%	5%
I	0.0	0%	0%
GUC	1.0	20%	40%
GUI	0.5	20%	20%
CUI	0.5	10%	10%
GUCUI	1.0	100%	100%

$$B(\cup A_i) \geq \sum_j B(A_j) - \sum_{j<k} B(A_j \cap A_k) + \ldots + (-1)^{n+k} B(\cap A_i)$$

Mőbius Representation

Source	Prob.	Panel 1		Panel 2	
		Belief	**m**	**Belief**	**m**
G	0.5	5%	5%	15%	15%
C	0.5	5%	5%	5%	5%
I	0.0	0%	0%	0%	0%
GUC	1.0	20%	10%	40%	20%
GUI	0.5	20%	15%	20%	5%
CUI	0.5	10%	15%	10%	5%
GUCUI	1.0	100%	60%	100%	50%

$$B(A) = \sum_{D \mid D \subseteq A} m(D)$$

For example, m(*G*) = B(*G*),
m(*G*) + m(*C*) + m(*G*U*C*) = B(*G*U*C*)

Mőbius Representation

Source	P	Panel 1 Belief	Panel 1 m	Panel 2 Belief	Panel 2 m	Combined Belief	Combined m
G	0.5	5%	5%	15%	15%	21%	21%
C	0.5	5%	5%	5%	5%	9%	9%
I	0.0	0%	0%	0%	0%	1%	1%
GUC	1.0	20%	10%	40%	20%	50%	20%
GUI	0.5	20%	15%	20%	5%	34%	12%
CUI	0.5	10%	15%	10%	5%	16%	6%
GUCUI	1.0	100%	60%	100%	50%	100%	31%

$$m_{1,2}(E) = \frac{\sum_{A \cap D = E} m_1(A) \cdot m_2(D)}{1 - c}$$

$$c = \sum_{A \cap D = 0} m_1(A) \cdot m_2(D)$$

Upper and Lower Probabilities

Consider two sets, $X(x_1, x_2)$ and $Y(y_1,y_2)$, with $\Sigma\, P_X(x_i) = \Sigma P_Y(y_i) = 1$

We seek $P_{X,Y}(x,y)$, which has four constraints:

$$P_{X,Y}(x_1,y_1)+P_{X,Y}(x_1,y_2)=P_X(x_1)$$

$$P_{X,Y}(x_2,y_1)+P_{X,Y}(x_2,y_2)=P_X(x_2)=1-P_X(x_1)$$

$$P_{X,Y}(x_1,y_1)+P_{X,Y}(x_2,y_1)=P_Y(y_1)$$

$$P_{X,Y}(x_1,y_2)+P_{X,Y}(x_2,y_2)=P_Y(y_2)=1-P_Y(y_1)$$

These are actually three independent equations for the four joint probabilities. Select $P_{X,Y}(x_1 y_1)$.

$$\max\{0, P_X(x_1) + P_Y(y_1)\} \le P_{X,Y}(x_1, y_1) \le \min\{P_X(x_1), P_Y(y_1)\}$$

Resulting in upper and lower bounds for any outcome, A, that consists of X,Y pairs:

$$\bar{P}(A) = \sup_D \sum_{X \in A} P_X(x)$$

$$\underline{P}(A) = \inf_D \sum_{X \in A} P_X(x)$$

Where D is the set of all consistent distributions

Conclusions

- We need to bridge the gap between classical mathematics and linguistic-based issues
- Generalized information theory has applications for linguistically-expressed, perceived risk
- Approaches include uncertainty measurements, fuzzy sets and generalized belief measures
- With broader approaches, mathematical theories can quantify risk factors, hazards, response, etc.